Unit 08

TRIGONOMETRY

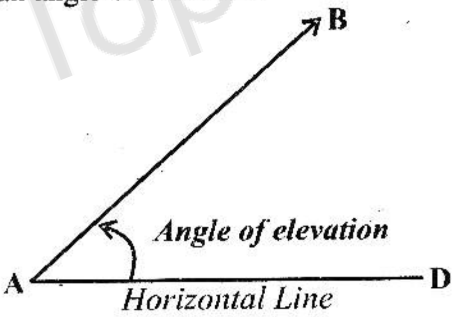
Trigonometry:
Trigonometry is the branch of mathematics which deals with sides and angles of a triangle as well as the relevant functions of its angles. The word trigonometry means triangle measurement.

Q.1 What is angle of elevation?

Ans. The angle made between the horizontal line $\overline{AD}$ and a line from the eye A to the object B (above D) is called angle of elevation B in fig <BAD is angle of elevation.

Or

The angle between the horizontal line through eye and a line from the eye to the object above the horizontal line is called an angle of elevation.



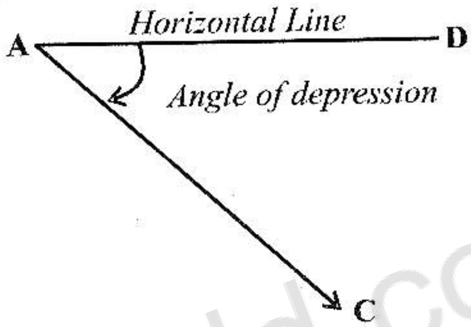
Q.2 What is angle of depression?

Ans. The angle made between the horizontal line $\overline{AD}$ and a line segment from the eye A to the object C (below D) is called angle of depression.

Or

The angle made between the horizontal line through eye and a line from the eye to the object below the horizontal line is called angle of depression.

In figure, < CAD is angle of depression



Q.3 Define Hypotenuse, Perpendicular and Base.

Ans.

Hypotenuse:

In a right angled triangle, the side facing right angle is called *hypotenuse.*

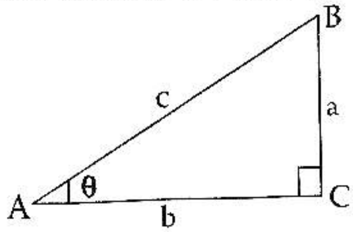
Perpendicular:

In a right angled triangle, the side facing the angle under consideration is called perpendicular.

Base:

In a right angled triangle, the side facing the common arm of right angle and the angle under consideration is called base.

Q.4 Take a right angled triangle ABC, show that $\sin^2\theta + \cos^2\theta = 1$.



Ans. By Pythagoras theorem, we have

$$a^2 + b^2 = c^2$$

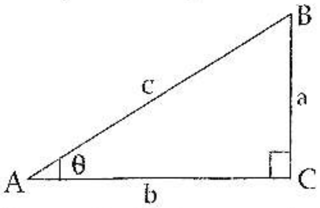
in figure, $\sin\theta = \dfrac{a}{c}$ $\cos\theta = \dfrac{b}{c}$

Dividing by c^2 both sides

$$\frac{a^2}{c^2}+\frac{b^2}{c^2}=\frac{c^2}{c^2}$$

$$\sin^2\theta+\cos^2\theta=1$$

Q.5 Take a right angled triangle ABC, show that $1+\tan^2\theta=\sec^2\theta$.



Ans. By Pythagoras theorem

$$a^2+b^2=c^2$$

Dividing by b^2

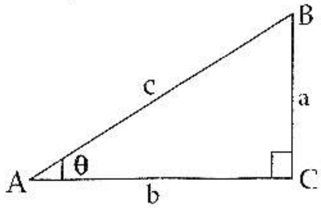
$$\frac{a^2}{b^2}+\frac{b^2}{b^2}=\frac{c^2}{b^2}$$

$$\frac{a^2}{b^2}+1=\frac{c^2}{b^2}$$

in figure, $\tan\theta=\frac{a}{b}$, $\sec\theta=\frac{c}{b}$

$$\tan^2\theta+1=\sec^2\theta$$

Q.6 Take a right angled triangle ABC, show that $1+\cot^2\theta=\operatorname{cosec}^2\theta$.



Ans. By Pythagoras theorem, we have

$$a^2+b^2=c^2$$

Dividing by a^2

$$\frac{a^2}{a^2}+\frac{b^2}{a^2}=\frac{c^2}{a^2}$$

$$1+(\frac{b}{a})^2+1=(\frac{c}{a})^2$$

In figure, $\cot\theta=\frac{b}{a}$

$$\cot\theta=\frac{c}{a}$$

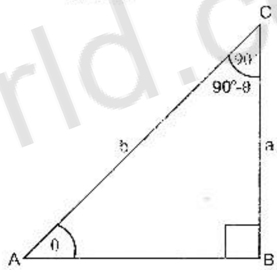
$$1+\cot^2\theta=\operatorname{cosec}^2\theta$$

Q.7 Prove that sin (90° - θ) = Cosθ

We consider a right-angled triangle ABC, in which $m\angle A=\theta$, $m\angle B=90^o$ then, $m\angle C=90^o-\theta$.

Using the trigonometric ratios of $\angle C$, we get

$$\sin(90^o-\theta)=\frac{m\overline{AB}}{m\overline{AC}}=\frac{c}{b}\text{...(i)}$$



$$\cos\theta=\frac{m\overline{AB}}{mAC}=\frac{c}{b}\text{.........(ii)}$$

From (i) and (ii), we get

$\sin(90^o-\theta)=\cos\theta$

Similarly, we have

$\cos(90^o-\theta)=\sin\theta$;

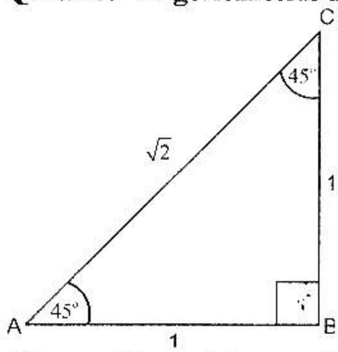
$\tan(90^o-\theta)=\cot\theta$;

$\cot(90^o-\theta)=\tan\theta$;

$\sec(90^o-\theta)=\operatorname{cosec}\theta$;

$\operatorname{cosec}(90^o-\theta)=\sec\theta$

Q.7 Find Trigonometric Ratios of 45°



We consider a right-anggled triangle ABC

with $m\angle ABC = 90^\circ$ and $m\overline{BC} = m\overline{AB} = 1$unit

Also $m\angle BAC = m\angle ACB = 45^\circ$ and $m\overline{AC} = \sqrt{2}$

Therefore;

$$\sin 45^\circ = \frac{m\overline{BC}}{m\overline{AC}} = \frac{1}{\sqrt{2}};$$

$$\operatorname{cosec} 45^\circ = \frac{m\overline{AC}}{m\overline{BC}} = \sqrt{2}$$

$$\cos 45^\circ = \frac{m\overline{AB}}{m\overline{AC}} = \frac{1}{\sqrt{2}};$$

$$\sec 45^\circ = \frac{m\overline{AC}}{m\overline{AB}} = \sqrt{2}$$

$$\tan 45^\circ = \frac{m\overline{BC}}{m\overline{AB}} = 1 \quad ;$$

$$\cot 45^\circ = \frac{m\overline{AB}}{m\overline{BC}} = 1$$

Q.8 Trigonometric Ratios of 30° and 60°

Consider a right angled triangle ABC, in which $m\angle B = 90^\circ$, $m\angle BAC = 60^\circ$, $m\angle ACB = 30^\circ$,

If $m\overline{AB} = 1$unit, then $m\overline{AC} = 2$ units.

By Pythagoras theorem, we have

$$|\overline{AB}|^2 + |\overline{BC}|^2 = |\overline{AC}|^2$$

Or $\quad (1)^2 + |\overline{BC}|^2 = (2)^2$

(Putting the values)

$$\Rightarrow \quad |\overline{BC}|^2 = 4 - 1 = 3$$

$$\Rightarrow \quad |\overline{BC}| = \sqrt{3}$$

In right angled triangle ABC,

$$\sin 30^\circ = \frac{m\overline{AB}}{m\overline{AC}} = \frac{1}{2} \Rightarrow \quad \operatorname{cosec} 30^\circ = 2$$

$$\cos 30^\circ = \frac{m\overline{BC}}{m\overline{AC}} = \frac{\sqrt{3}}{2} \Rightarrow \sec 30^\circ = \frac{2}{\sqrt{3}}$$

$$\tan 30^0 = \frac{m\overline{AB}}{m\overline{BC}} = \frac{1}{\sqrt{3}} \Rightarrow \quad \cot 30^0 = \sqrt{3}$$

Again in the right angled triangle ABC,

$$\sin 60^\circ = \frac{m\overline{BC}}{m\overline{AC}} = \frac{\sqrt{3}}{2} \Rightarrow \quad \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$$

$$\cos 60^\circ = \frac{m\overline{AB}}{m\overline{AC}} = \frac{1}{2} \Rightarrow \sec 60^\circ = 2$$

$$\tan 60^0 = \frac{m\overline{BC}}{m\overline{AB}} = \sqrt{3} \Rightarrow \quad \cot 60^0 = \frac{1}{\sqrt{3}}$$

These results in the form of a table can be written as:

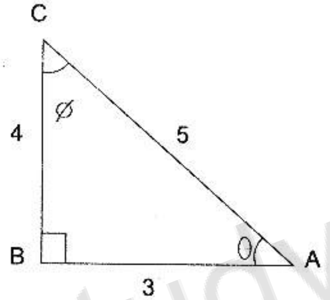
Q	0°	30°	45°	60°	90°
Sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

Exercise 8.1

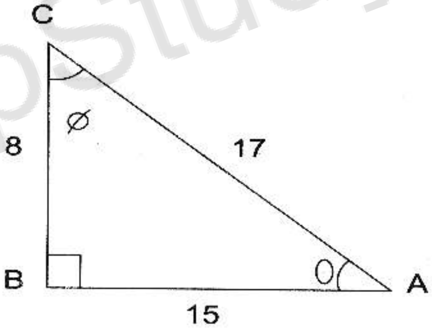
Q.1 For each of the following right-angled triangles, find the trigonometric ratios:

i. $\sin\theta$	**ii.** $\cos\theta$
iii. $\tan\theta$	**iv.** $\sec\theta$
v. $\text{cosec}\,\theta$	**vi.** $\cot\phi$
vii. $\tan\phi$	**viii.** $\sin\phi$
ix. $\sec\phi$	**x.** $\cos\phi$

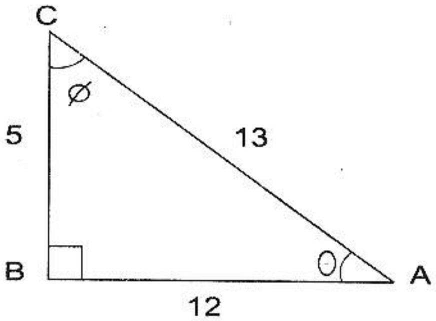
a)



b)



c)



Sol:

Consider triangle ABC in fig (a)

i) $Sin\theta = \frac{4}{5}$

ii) $Cos\theta = \frac{3}{5}$

iii) $tan\theta = \frac{4}{3}$

iv) $\sec\theta = \frac{5}{3}$

v) $Co\sec\theta = \frac{5}{4}$

vi) $Cot\phi = \frac{4}{3}$

vii) $\tan\phi = \frac{3}{4}$

viii) $\sin\phi = \frac{3}{5}$

ix) $Cos\phi = \frac{4}{5}$

Consider fig. (b) then:

i) $sin\theta = \frac{8}{17}$

ii) $cos\theta = \frac{15}{17}$

iii) $\tan\theta = \frac{8}{15}$

iv) $\sec\theta = \frac{17}{15}$

v) $co\sec\theta = \frac{17}{8}$

vi) $cot\theta = \frac{15}{8}$

vii) $\tan\phi = \frac{15}{8}$

viii) $\sin\phi = \frac{15}{7}$

ix) $sec\phi = \frac{17}{8}$

x) $cos\phi = \frac{8}{17}$

Consider fig. (c) then:

i) $sin\theta = \frac{5}{13}$

ii) $cos\theta = \frac{12}{13}$

iii) $\tan\theta = \frac{5}{12}$

iv) $\sec\theta = \frac{13}{12}$

v) $co\sec\theta = \frac{13}{5}$

vi) $cot\theta = \frac{12}{5}$

vii) $\tan\phi = \frac{12}{5}$

viii) $\sin\phi = \frac{12}{13}$

ix) $\sec\phi = \frac{13}{5}$

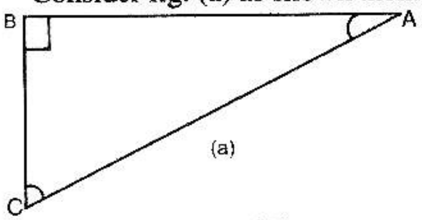
x) $cos\phi = \frac{5}{13}$

Q.2 For each of the following right-angled triangles find the trigonometric ratios for which

i. sin m∠A	**ii. cos m∠A**
iii. tan m∠A	**iv. sin m∠C**
v. cos m∠C	**vi. tan m∠C**

Sol:

Consider fig. (a) as shown then:



i) $Sin\, m\angle A = \frac{m\overline{BC}}{m\overline{AC}}$

ii) $Cos\, m\angle A = \frac{m\overline{AB}}{m\overline{AC}}$

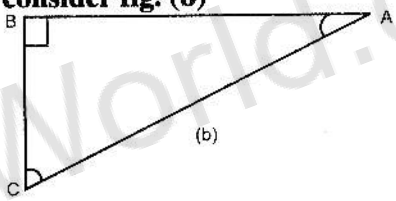
iii) $\tan m\angle A = \frac{m\overline{BC}}{m\overline{AB}}$

iv) $Sin\, m\angle C = \frac{m\overline{AB}}{m\overline{AC}}$

v) $Cos\, m\angle C = \frac{m\overline{BC}}{m\overline{AC}}$

vi) $\tan m\angle C = \frac{m\overline{AB}}{m\overline{BC}}$

Now consider fig. (b)



Then:

i) $\sin m\angle A = \frac{m\overline{BC}}{m\overline{AC}}$

ii) $\cos\, m\angle A = \frac{m\overline{AB}}{m\overline{AC}}$

iii) $\tan\, m\angle A = \frac{m\overline{BC}}{m\overline{AB}}$

iv) $\sin\, m\angle C = \frac{m\overline{AB}}{m\overline{AC}}$

v) $\cos\, m\angle C = \frac{m\overline{BC}}{m\overline{AC}}$

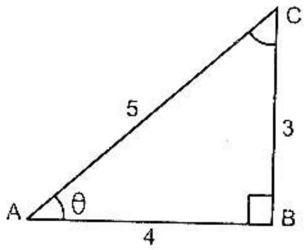
vi) $\tan\, m\angle C = \frac{m\overline{AB}}{m\overline{BC}}$

Q.3 Considering the adjoining triangle ABC verify that

i.	**$\sin\theta$ cosecθ**	**= 1**
ii.	**$\cos\theta$ $\sec\theta$**	**= 1**
iii.	**$\tan\theta$ $\cot\theta$**	**= 1**

Sol:

i) Consider the right angle triangle as shown:



Then $Sin\theta = \frac{3}{5}$

and $\cos ec\theta = \frac{1}{\sin\theta} = \frac{5}{3}$

Now $\sin\theta \cos ec\theta = \frac{3}{5}.\frac{5}{3}$

$\sin\theta \cos ec\theta = 1$

ii) $Cos\theta = \frac{4}{5}$

and $Sec\theta = \frac{1}{Cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

Now

$Cos\theta.Sec\theta = \frac{4}{5}.\frac{5}{4}$

$= 1$

So $Cos\theta.Sec\theta = 1$

iii) $\tan\theta = \frac{3}{4}$

and $Cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$

So $\tan\theta.Cot\theta = \frac{3}{4}.\frac{4}{3} = 1$

Q.4 Fill in the blanks.

i) $Sin30^o = Sin(90^o - 60^o) = Cos$.............

Sol: 60^o

ii) $Cos30^o = Cos(90^o - 60^o) = Sin$.............

Sol: 60^o

iii) $\tan 60^o = \tan(90^o - 30^o) = Cot$.............

Sol: 30^o

iv) $Sin60^o = Sin(90^o - 30^o) = Cos$.............

Sol: 30^o

v) $Cos60^o = Cos(90^o - 30^o) = Sin$.............

Sol: 30^o

vi) $Sin45^o = Sin(90^o - 45^o) = Cos$.............

Sol: 45^o

vii) $\tan 45^o = \tan(90^o - 45^o) = Cot$.............

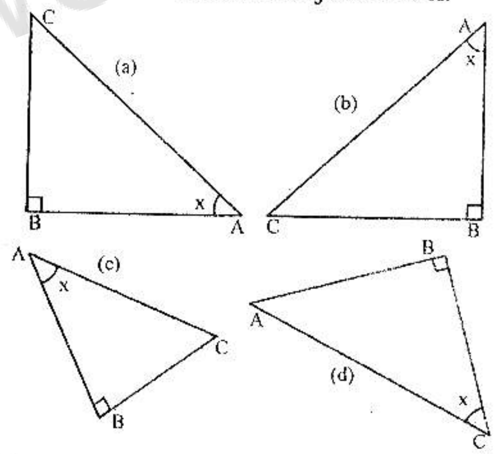
Sol: 45^o

viii) $Cos45^o = Cos(90^o - 45^o) = Sin$.............

Sol: 45^o

Q.5 For each of the following right angled triangles, name

i. The hypotenuse
ii. The side opposite to x.
iii. The side adjacent to x.



Sol:

Consider fig. (a)

i) The hypotenuse is $\overline{AC}$

ii) The side opposite to x is $\overline{BC}$

iii) The side adjacent to x is $\overline{AB}$

Consider fig. (b)

i) The hypotenuse is $\overline{AC}$

ii) The side opposite to x is $\overline{BC}$

iii) The side adjacent to x is $\overline{AB}$

Consider fig. (c)

i) The hypotenuse is $\overline{AC}$

ii) The side opposite to x is $\overline{BC}$

iii) The side adjacent to x is $\overline{AB}$

Consider fig. (d)

i) The hypotenuse is $\overline{AC}$

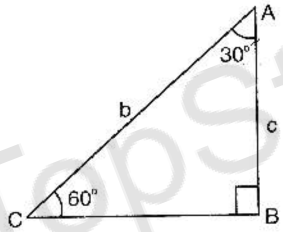
ii) The side opposite to x is $\overline{AB}$

iii) The side adjacent to x is $\overline{BC}$

Q.6 If in a right angled triangle ABC, the angle B is 90^o and C is an acute angle of 60^o. Also ($\sin m\angle C = \frac{c}{b}$) then find the following trigonometric ratios.

i) $\frac{m\overline{BC}}{m\overline{AB}}$

Sol: Consider a right angled triangle as show:



Where $m\angle B = 90^o$

$m\angle C = 60^o$

Also $Sin\, m\angle C = \frac{c}{b}$

So $m\overline{AB} = c$

$m\overline{AC} = b$

So $m\overline{BC} = \sqrt{b^2 - c^2}$

i) $\frac{m\overline{BC}}{m\overline{AB}}$

Sol: The above ratio is:

$$\frac{m\overline{BC}}{m\overline{AB}} = \frac{\sqrt{b^2-c^2}}{c}$$

ii) $cos60^o$

Sol: $cos60^o = \frac{m\overline{BC}}{m\overline{AC}}$

$$= \frac{\sqrt{b^2-c^2}}{b}$$

iii) $\tan 60^o$

Sol: $\tan 60^o = \frac{m\overline{AB}}{m\overline{BC}}$

$$= \frac{c}{\sqrt{b^2-c^2}}$$

iv) $\sec 60^o$

Sol: $sec60^o = \frac{m\overline{AC}}{m\overline{BC}}$

$$= \frac{b}{\sqrt{b^2-c^2}}$$

v) $co\sec 60^o$

Sol: $Co\sec 60^o = \frac{m\overline{AC}}{m\overline{AB}}$

$$= \frac{b}{c}$$

vi) $Cot60^o$

Sol: $Cot60^o = \frac{m\overline{BC}}{m\overline{AB}}$

$$= \frac{\sqrt{b^2-c^2}}{c}$$

vii) $Sin30^o$

Sol: $Sin30^o = \frac{m\overline{BC}}{m\overline{AC}}$

$$= \frac{\sqrt{b^2-c^2}}{b}$$

viii) $Cos30^o$

Sol: $Cos30^o = \frac{m\overline{AB}}{m\overline{AC}}$

$$= \frac{c}{b}$$

ix) $\tan 30^o$

Sol: $\tan 30^o = \frac{m\overline{BC}}{m\overline{AB}}$

$= \frac{\sqrt{b^2-c^2}}{c}$

x) $\sec 30^o$

Sol: $\sec 30^o = \frac{m\overline{AC}}{m\overline{AB}}$

$= \frac{b}{c}$

xi) $Co\sec 30^o$

Sol: $Co\sec 30^o = \frac{m\overline{AC}}{m\overline{BC}}$

$= \frac{b}{\sqrt{b^2-c^2}}$

xii) $Cot 30^o$

Sol: $Cot 30^o = \frac{m\overline{AB}}{m\overline{BC}}$

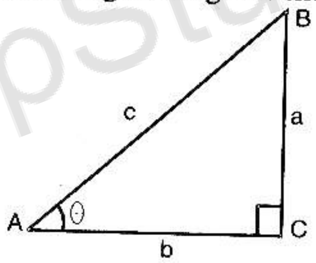
$= \frac{c}{\sqrt{b^2-c^2}}$

EXERCISE 8.2

Q.1 If $\sin\theta = \frac{2}{3}$, then find the remaining trigonometric ratios when θ lies in first quadrant.

Sol:

Consider a right triangle as shown:



Here $Sin\theta = \frac{2}{3} = \frac{a}{c}$

$\Rightarrow \quad a = 2$

$c = 3$

Hence

$b = \sqrt{c^2 - a^2}$

$b = \sqrt{3^2 - 2^2}$

$= \sqrt{9-4}$

$b = \sqrt{5}$

So trigonometric ratios are:

$Sin\theta = \frac{a}{c} = \frac{2}{3}$

$Cos\theta = \frac{b}{c} = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{a}{b} = \frac{2}{\sqrt{5}}$

$Cot\theta = \frac{b}{a} = \frac{\sqrt{5}}{2}$

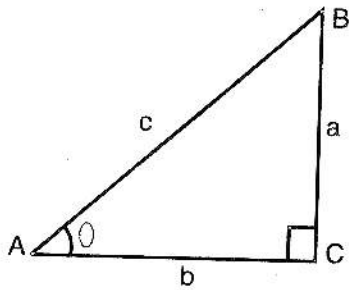
$Sec\theta = \frac{c}{b} = \frac{3}{\sqrt{5}}$

$Cosec\theta = \frac{c}{a} = \frac{3}{2}$

Q.2 If $\cos\theta = \frac{3}{5}$, then find the remaining trigonometric ratios when θ lies in first quadrant.

Sol:

Consider a right angled triangle as shown:



Here $Cos\theta = \frac{3}{5} = \frac{b}{c}$

$\Rightarrow \quad b = 3$

$c = 5$

Hence

$\therefore \quad a = \sqrt{c^2 - b^2}$

$a = \sqrt{5^2 - 3^2}$

$= \sqrt{25 - 9}$

$= \sqrt{16}$

$a = 4$

So trigonometric ratios are:

$Sin\theta = \frac{a}{c} = \frac{4}{5}$

$Cos\theta = \frac{b}{c} = \frac{3}{5}$

$\tan\theta = \frac{a}{b} = \frac{4}{3}$

$Cot\theta = \frac{b}{a} = \frac{3}{4}$

$Sec\theta = \frac{c}{b} = \frac{5}{3}$

$Co\sec\theta = \frac{c}{a} = \frac{5}{4}$

Q.3 Prove that

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Sol:

L.H.S $= (\sin\theta + \cos\theta)^2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$\therefore \quad \sin^2\theta + \cos^2\theta = 1$

$= 1 + 2\sin\theta\cos\theta$

$=$ R.H.S

So

L.H.S = R.H.S

Q.4. $\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

Sol: L.H.S $= \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\frac{\sin\theta}{\cos\theta}}$

$= \frac{1}{\tan\theta}$

$=$ R.H.S

So

L.H.S = R.H.S

Q.5 $\frac{Sin\theta}{Co\sec\theta} + \frac{Cos\theta}{Sec\theta} = 1$

Sol: L.H.S $= \frac{Sin\theta}{Co\sec\theta} + \frac{Cos\theta}{Sec\theta}$

$= \frac{Sin\theta}{\frac{1}{Sin\theta}} + \frac{Cos\theta}{\frac{1}{Cos\theta}}$

$= Sin^2\theta + Cos^2\theta$

$= 1$

$=$ R.H.S

So

L.H.S = R.H.S

Q.6 $\mathbf{Cos^2\theta - Sin^2\theta = 2Cos^2\theta - 1}$

Sol: L.H.S $= Cos^2\theta - Sin^2\theta$

$= Cos^2\theta - (1 - Cos^2\theta) \quad (\therefore Sin^2\theta = 1 - Cos^2\theta)$

$= Cos^2\theta - 1 + Cos^2\theta$

$= 2Cos^2\theta - 1$

$=$ R.H.S

So

L.H.S = R.H.S

Q.7 $\mathbf{Cos^2\theta - Sin^2\theta = 1 - 2Sin^2\theta}$

Sol: L.H.S $= Cos^2\theta - Sin^2\theta$

$= 1 - \sin^2\theta - \sin^2\theta$

$(\therefore \cos^2\theta = 1 - \sin^2\theta)$

$= 1 - 2\sin^2\theta$

$=$ R.H.S

So

L.H.S = R.H.S

Q.8 $\mathbf{(1 + \sin\theta)(1 - \sin\theta) = \frac{1}{\sec^2\theta}}$

Sol: L.H.S $= (1 + \sin\theta)(1 - \sin\theta)$

$\therefore$ $(a + b)(a - b) = a^2 - b^2$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

$\therefore$ $\sec^2\theta = \dfrac{1}{\cos^2\theta}$

$= \dfrac{1}{\sec^2\theta}$

$=$ R.H.S

So

L.H.S $=$ R.H.S

Q.9 $(1 - \cos\theta)(1 + \cos\theta) = \dfrac{1}{\cos ec^2\theta}$

Sol: L.H.S $= (1 - \cos\theta)(1 + \cos\theta)$

$= 1 - \cos^2\theta$

$\therefore$ $(a + b)(a - b) = a^2 - b^2$

$= \sin^2\theta$

$= \dfrac{1}{\cos ec^2\theta}$

$(\sin\theta = \dfrac{1}{\cos\theta})$

$= \dfrac{1}{Co\sec^2\theta}$

$=$ R.H.S

So

L.H.S $=$ R.H.S

Q.10 $(\sec\theta - 1)(\sec\theta + 1) = \tan^2\theta$

Sol.

L.H.S $= (\sec\theta - 1)(\sec\theta + 1)$

$= \sec^2\theta - 1$

$= 1 + \tan^2\theta - 1$

$\left(\because \sec^2\theta = 1 + \tan^2\theta\right)$

$= \tan^2\theta$

$=$ R.H.S

So

L.H.S $=$ R.H.S

Q.11 $(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1) = \cot^2\theta$

Sol:

L.H.S $= (\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)$

$= \text{cosec}^2\theta - 1$

$= 1 + \cot^2\theta - 1$

$\left(\because Co\sec^2\theta = 1 + Cot^2\theta\right)$

$= Cot^2\theta$

$=$ R.H.S

So

L.H.S $=$ R.H.S

Q.12 $\dfrac{1 - \sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1 + \sin\theta}$

Sol:

L.H.S $= \dfrac{1 - \sin\theta}{\cos\theta}$

$= \dfrac{1 - \sin\theta}{\cos\theta} \times \dfrac{1 + \sin\theta}{1 + \sin\theta}$

$= \dfrac{1 - \sin^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{cos^2\theta}{\cos\theta(1 + \sin\theta)}$

$(\therefore\ 1 - \sin^2\theta = \cos^2\theta)$

$= \dfrac{cos\theta}{1 + \sin\theta}$

$=$ R.H.S

L.H.S $=$ R.H.S

Q.13 $\dfrac{\sin\theta}{1 - \cos\theta} = \dfrac{1 + \cos\theta}{\sin\theta}$

Sol:

L.H.S $= \dfrac{\sin\theta}{1 - \cos\theta}$

$= \dfrac{\sin\theta}{1 - \cos\theta} \times \dfrac{1 + \cos\theta}{1 + \cos\theta}$

$= \dfrac{\sin\theta(1 + \cos\theta)}{1 - \cos^2\theta}$

$= \dfrac{\sin\theta(1 + \cos\theta)}{\sin^2\theta}$

$(\therefore\ 1 - \cos^2\theta = \sin^2\theta)$

$= \dfrac{1 + \cos\theta}{\sin\theta}$

R.H.S. $=$ L.H.S.

Q.14 $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Sol:

L.H.S $= 2\cos^2\theta - 1$

$= 2(1-\sin^2\theta) - 1$

$(\therefore \ \cos^2\theta = 1 - \sin^2\theta)$

$= 2 - 2\sin^2\theta - 1$

$= 1 - 2\sin^2\theta$

$=$ R.H.S

So

L.H.S $=$ R.H.S

Q.15 $\cot\theta . \sin\theta = \dfrac{1}{\sec\theta}$

Sol:

L.H.S $= \cot\theta \ . \sin\theta$

$= \dfrac{\cos\theta}{\sin\theta}.\sin\theta$

$= \cos\theta$

$\therefore \ = \dfrac{1}{\sec\theta} \quad (\therefore \ \cos\theta = \dfrac{1}{\sec\theta})$

$=$ R.H.S

So

L.H.S $=$ R.H.S

Q.16 $\sin\theta\ (\text{cosec}\,\theta - \sin\theta) = \dfrac{1}{\sec^2\theta}$

Sol:

L.H.S $= \sin\theta\ (\text{cosec}\,\theta - \sin\theta)$

$= \sin\theta\left(\dfrac{1}{\sin\theta} - \sin\theta\right)$

$= \sin\theta\left(\dfrac{1-\sin^2\theta}{\sin\theta}\right)$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

$\therefore \ \cos^2\theta = \dfrac{1}{\sec^2\theta}$

$\therefore \ = \dfrac{1}{\sec^2\theta}$

$=$ R.H.S

So

L.H.S $=$ R.H.S

Q.17 $\cos\theta(\tan\theta + \cot\theta) = \text{cosec}\,\theta$

Sol:

L.H.S $= \cos\theta(\tan\theta + \cot\theta)$

$= \cos\theta\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)$

$= \cos\theta\left(\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$

$= \dfrac{1}{\sin\theta} \quad \therefore\ \sin^2\theta + \cos^2\theta = 1$

$= \cos ec\theta$

$=$ R.H.S

So

L.H.S $=$ R.H.S

Q.18 $\dfrac{\tan\theta + \cot\theta}{\cos ec\theta} = \sec\theta$

Sol:

L.H.S $= \dfrac{\tan\theta + \cot\theta}{\cos ec\theta}$

$= \dfrac{1}{\cos ec\theta}(\tan\theta + \cot\theta)$

$= \sin\theta[\tan\theta + \cot\theta]$

$\therefore \ \dfrac{1}{\cos ec\theta} = \sin\theta$

$= \sin\theta\left[\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right]$

$= \sin\theta\left[\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right]$

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

$=$ R.H.S

So L.H.S $=$ R.H.S

Q.19 $\dfrac{\cos ec\theta - \sec\theta}{\cos ec\theta + \sec\theta} = \dfrac{1-\tan\theta}{1+\tan\theta}$

Sol:

L.H.S.

$= (\cos ec\theta - \sec\theta) \div (\cos ec\theta + \sec\theta)$

$$= \left(\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right)\div\left(\frac{1}{\sin\theta}+\frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\cos\theta-\sin\theta}{\sin\theta.\cos\theta}\right)\div\left(\frac{\cos\theta+\sin\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\cos\theta-\sin\theta}{\sin\theta.\cos\theta}\times\frac{\sin\theta.\cos\theta}{\sin\theta+\cos\theta}$$

$$= \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

dividing numerator and denominator by $\cos\theta$

$$= \frac{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{1-\tan\theta}{1+\tan\theta}$$

So

L.H.S = R.H.S

Q.20 $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$

Sol:

$$\text{L.H.S} = \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

= R.H.S

So

L.H.S = R.H.S

Q.21 $\sec\theta = \sqrt{1+\tan^2\theta}$

Sol:

$$\text{L.H.S} = \sqrt{1+\tan^2\theta}$$

$$= \sqrt{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \sqrt{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \sqrt{\frac{1}{\cos^2\theta}}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

= L.H.S

So

L.H.S = R.H.S

Q.22 $\mathbf{cosec}\,\theta = \sqrt{1+\cot^2\theta}$

Sol:

$$\text{R.H.S} = \sqrt{1+\cot^2\theta}$$

$$= \sqrt{1+\frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \sqrt{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$$

$$= \sqrt{\frac{1}{\sin^2\theta}}$$

$$= \frac{1}{\sin\theta}$$

$$= \operatorname{cosec}\theta$$

= L.H.S

So

L.H.S = R.H.S

EXERCISE 8.3

Q.1 Find the value of the following trigonometric ratios without using the calculator.

i. **$\sin 30^\circ$** **Sol:** $\frac{1}{2}$

ii. **$\cos 30^\circ$** **Sol:** $\frac{\sqrt{3}}{2}$

iii. **$\tan 30^\circ$** **Sol:** $\frac{1}{\sqrt{3}}$

iv. **$\tan 60^\circ$** **Sol:** $\sqrt{3}$

v. **$\sec 60^\circ$** **Sol:** 2

vi. **$\cos 60^\circ$** **Sol:** $\frac{1}{2}$

vii. **$\cot 60^\circ$** **Sol:** $\frac{1}{\sqrt{3}}$

viii. **$\sin 60^\circ$** **Sol:** $\frac{\sqrt{3}}{2}$

ix. **$\sec 30^\circ$** **Sol:** $\frac{2}{\sqrt{3}}$

x. **$\operatorname{cosec} 30^\circ$** **Sol:** 2

xi. **$\sin 45^\circ$** **Sol:** $\frac{1}{\sqrt{2}}$

xii. **$\cos 45^\circ$** **Sol:** $\frac{1}{\sqrt{2}}$

Q.2 Find the values of the each part:

i. $2\sin 60^\circ \cos 60^\circ$

Sol: $2\sin 60^\circ \cos 60^\circ$

$$= 2.\frac{\sqrt{3}}{2}.\frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}$$

ii. $2\cos 60^\circ \sin 60^\circ$

Sol: $2\cos 60^\circ \sin 60^\circ$

$$= 2.\frac{1}{2}.\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}$$

iii. $2\sin 45^\circ + 2\cos 45^\circ$

Sol: $2\sin 45^\circ + 2\cos 45^\circ$

$$= 2.\frac{1}{\sqrt{2}} + 2.\frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}$$

$$= \frac{2+2}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}}$$

iv. $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Sol: $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{1}{2}$$

$$= \frac{\left(\sqrt{3}\right)^2}{4} + \frac{1}{4}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{3+1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

v. $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

Sol: $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$= \frac{1}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}.\frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= 0$$

vi. $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

Sol: $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} - \frac{1}{2}.\frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4}$$
$$= \frac{3-1}{4}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

vii. $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

Ans: $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$
$$= \frac{\sqrt{3}+\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

viii. $\tan 30^\circ \cot 30^\circ + 1$

Sol: $\tan 30^\circ \cot 30^\circ + 1$

$$= \frac{1}{\sqrt{3}} . \sqrt{3} + 1$$
$$= 1 + 1$$
$$= 2$$

Q.3 If $\sin 45^\circ$ and $\cos 45^\circ$ equal to $\frac{1}{\sqrt{2}}$ each, then find the values of followings:

i. $2\sin 45^\circ + 2\cos 45^\circ$

Sol: $2\sin 45^\circ + 2\cos 45^\circ$

$$= 2.\frac{1}{\sqrt{2}} + 2.\frac{1}{\sqrt{2}}$$
$$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}$$
$$= \frac{4}{\sqrt{2}}$$

ii. $3\cos 45^\circ + 4\sin 45^\circ$

Sol: $3\cos 45^\circ + 4\sin 45^\circ$

$$= 3.\frac{1}{\sqrt{2}} + 4.\frac{1}{\sqrt{2}}$$
$$= \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{2}}$$
$$= \frac{3+4}{\sqrt{2}}$$
$$= \frac{7}{\sqrt{2}}$$

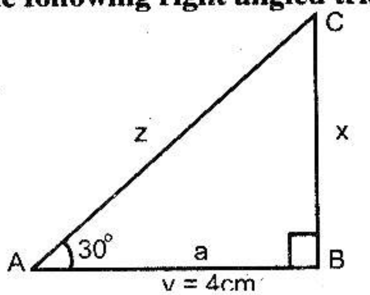
iii. $5\cos 45^\circ - 3\sin 45^\circ$

Sol: $5\cos 45^\circ - 3\sin 45^\circ$

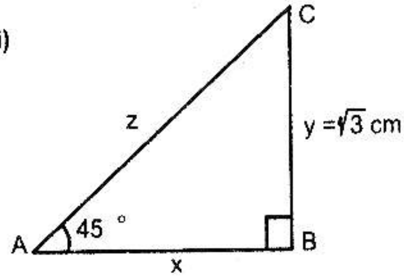
$$= 5.\frac{1}{\sqrt{2}} - 3.\frac{1}{\sqrt{2}}$$
$$= \frac{5}{\sqrt{2}} - \frac{3}{\sqrt{2}}$$
$$= \frac{2}{\sqrt{2}}$$

EXERCISE 8.4

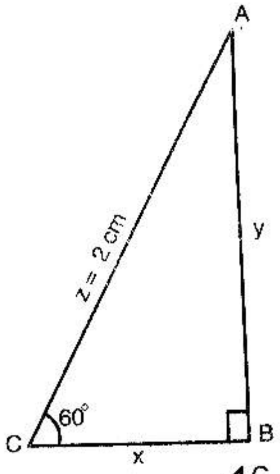
Q.1 Find the value of x, y and z from the following right angled triangles.

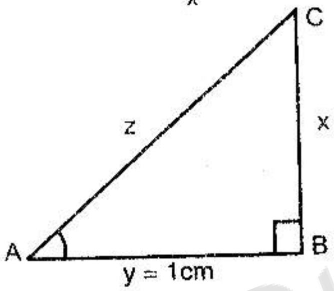


(ii)



(iii)





(iv)

Sol:

Consider fig. (i)

From fig. (i)

$$\tan 30^o = \frac{x}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{4}$$

$$\Rightarrow \frac{x}{4} = \frac{1}{\sqrt{3}}$$

$$x = \frac{4}{\sqrt{3}}$$

By Pythagoras Theorem

$$z^2 = x^2 + y^2$$

$$= \left(\frac{4}{\sqrt{3}}\right)^2 + (4)^2$$

$$= \frac{16}{3} + 16$$

$$= \frac{16+48}{3}$$

$$z^2 = \frac{64}{3}$$

$$z = \frac{8}{\sqrt{3}} \text{ Ans.}$$

Now consider fig. (ii)

The given triangle is isosceles:

So $x = y$

Hence $x = \sqrt{3}$

By Pythagoras Theorem

$$z^2 = x^2 + y^2$$

$$= \left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2$$

$$= 3 + 3$$

$$z^2 = 6$$

$$z = \sqrt{6}$$

Consider fig. (iii)

Here $Sin60^o = \dfrac{y}{z}$

$$y = z.Sin60^o$$

$$y = z.\frac{\sqrt{3}}{2}$$

$$y = 2\left(\frac{\sqrt{3}}{2}\right)$$

$$y = \sqrt{3} \text{ cm}$$

Now from fig.

$$x^2 = z^2 - y^2$$

$$= (2)^2 - \left(\sqrt{3}\right)^2$$

$$= 4 - 3$$

$$x^2 = 1 \quad \Rightarrow \quad x = 1$$

Consider fig (iv)

$$\tan 45^o = \frac{x}{y}$$

$$1 = \frac{x}{1}$$

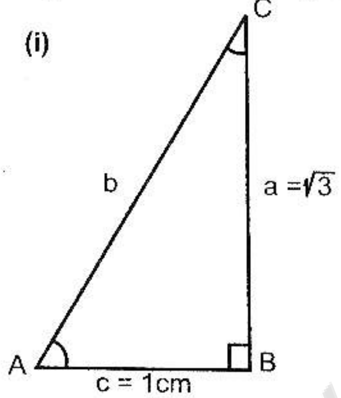
$$\Rightarrow x = 1cm$$

Now by Pythagoras Theorem:

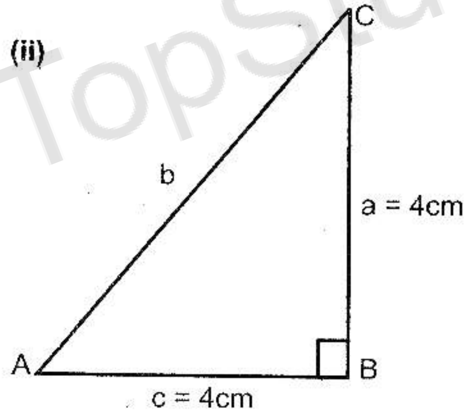
$$z^2 = x^2 + y^2$$
$$= (1)^2 + (1)^2$$
$$= 1 + 1$$
$$z^2 = 2$$
or $z = \sqrt{2}$

Q.2 Find the unknown side and angles of the following triangles.

(i)



(ii)



Sol:

Consider fig (i)

Here $a = \sqrt{3}cm$

$c = 1cm$

$m\angle B = 90^o$

$b = ?$

$m\angle A = ?$

$m\angle C = ?$

By Pythagoras Theorem

$$b^2 = a^2 + c^2$$
$$= \left(\sqrt{3}\right)^2 + (1)^2$$
$$= 3 + 1$$
$$b^2 = 4$$
$$\boxed{b = 2\text{ cm}}$$
$$b = 2\text{ cm}$$

Now $\tan m\angle A = \dfrac{a}{c}$

$$= \frac{\sqrt{3}}{1}$$

$\tan m\angle A = \sqrt{3}$

So

$$\boxed{m\angle A = 60^o}$$

Now

$$m\angle C = 90^o$$
$$= 90^o - 60^o$$
$$\boxed{m\angle C = 30^o}$$

Now

Consider fig (ii)

$m\angle B = 90^o$

$a = 4cm$

$c = 4cm$

$b = ?$

$m\angle A = ?$

$m\angle C = ?$

Now

$$\tan m\angle A = \frac{a}{c}$$
$$= \frac{4}{4}$$
$$\tan m\angle A = 1$$

So $\boxed{m\angle A = 45^o}$

Hence

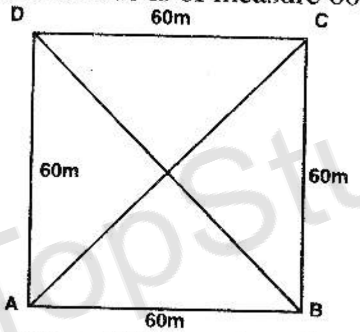
$$m\angle C = 90^o - m\angle A$$
$$= 90^o - 45^0$$
$$\boxed{m\angle C = 45^o}$$

Now

$$b^2 = a^2 + c^2$$
$$= (4)^2 + (4)^2$$
$$= 16 + 16$$
$$b^2 = 32$$
$$b = \sqrt{32}$$
$$= \sqrt{16 \times 2}$$
$$b = 4\sqrt{2}cm$$

Q.3 Each side of a square field is 60m long. Find the lengths of the diagonals of the field.

Sol: Consider a square field ABCD.
Here each side is of measure 60m.



Here AC and BD are two diagonals of square field.

Applying Pythagoras theorem on right angled ΔABC.

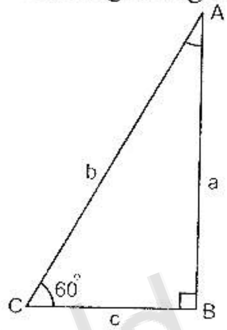
$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$
$$= (60)^2 + (60)^2$$
$$= 3600 + 3600$$
$$(\overline{AC})^2 = 7200$$
$$\overline{AC} = \sqrt{7200}$$
$$= \sqrt{3600 \times 2}$$
$$\overline{AC} = 60\sqrt{2}\ \text{m}$$

Also

$\overline{BD} = 60\sqrt{2}$ m are length of diagonals

Q.4 Solve the following triangles when m ∠ B = 90°, m ∠ C = 60°, C = $3\sqrt{3}$ cm.

Sol. Consider a right angled ΔABC.



$$m\angle B = 90^o$$
$$m\angle C = 60^o$$
$$c = 3\sqrt{3}cm$$
$$m\angle A = ?$$
$$a = ?$$
$$b = ?$$
$$m\angle A = 90^o - m\angle C$$
$$= 90^o - 60^o$$
$$m\angle A = 30^0$$

Now

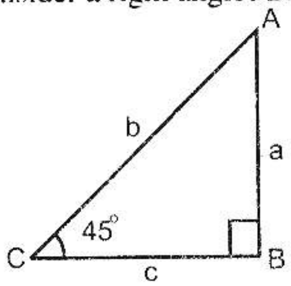
$$\tan m\angle A = \frac{c}{a}$$
$$\tan 60^o = \frac{c}{a}$$
$$\sqrt{3} = \frac{3\sqrt{3}}{a}$$
$$a = \frac{3\sqrt{3}}{\sqrt{3}} = 3cm$$

Now by Pythagoras theorem

$$b^2 = a^2 + c^2$$
$$= (3)^2 + (3\sqrt{3})^2$$
$$b^2 = 9 + 27$$
$$b^2 = 36$$
$$b = 6cm$$

Q.5 **$m\angle C = 45^\circ$, $a = 8cm$ and $m\angle B = 90^\circ$**

Sol. Consider a right angled ΔABC.



Here

$$m\angle C = 45^0$$
$$a = 8cm$$
$$m\angle B = 90^0$$
$$m\angle A = ?$$
$$b = ?$$
$$c = ?$$

Now

$$m\angle A = 90^0 - m\angle C$$
$$= 90^0 - 45^0$$
$$m\angle A = 45^0$$

Now

$$\tan m\angle A = \frac{a}{c}$$
$$\tan 45^\circ = \frac{8}{c}$$
$$1 = \frac{8}{c}$$
$$c = 8cm$$

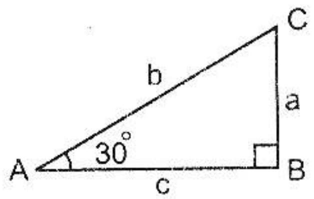
Now by Pythagoras theorem

$$b = \sqrt{a^2+c^2}$$
$$= \sqrt{(8)^2+(8)^2}$$
$$b = \sqrt{64+64}$$
$$= \sqrt{128}$$
$$= \sqrt{64\times 2}$$
$$b = 8\sqrt{2}cm$$

Q.6 **$m\angle A = 30^\circ$, $m\angle B = 90^\circ$, $b = 6cm$**

Sol. Consider a right angled ΔABC.

Here



$$m\angle A = 30^0$$
$$m\angle B = 90^0$$
$$b = 6cm$$
$$m\angle C = ?$$
$$a = ?$$
$$c = ?$$

Now

$$m\angle C = 90^0 - m\angle A$$
$$= 90^0 - 30^0$$
$$m\angle C = 60^0$$

In ΔABC,

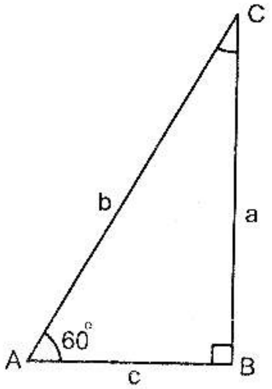
$$Sin\, m\angle A = \frac{a}{b}$$
$$Sin\, 30^0 = \frac{a}{6}$$
$$\Rightarrow a = 6 Sin\, 30^0$$
$$= 6\times\frac{1}{2}$$
$$a = 3cm$$

Now by Pythagoras theorem

$$c^2 = b^2 - a^2$$
$$= (6)^2 - (3)^2$$
$$= 36 - 9$$
$$c^2 = 27$$
$$c = \sqrt{27}$$
$$= \sqrt{9\times 3}$$
$$c = 3\sqrt{3}cm$$

Q.7 **$m\angle A = 60^\circ$, $c = 4cm$, $m\angle B = 90^\circ$**

Sol. Consider a right angled ΔABC.



Here

$m\angle A = 60^0$

$m\angle B = 90^0$

$m\angle C = ?$

$c = 4cm$

$a = ?$

$b = ?$

Now

$m\angle C = 90^0 - m\angle A$

$= 90^0 - 60^0$

$m\angle C = 30^0$

Now in ΔABC,

$\tan m\angle A = \frac{a}{c}$

$\tan 60^0 = \frac{a}{4}$

$\sqrt{3} = \frac{a}{4}$

$\Rightarrow a = 4\sqrt{3}cm$

$a = 6.92cm$

Now by Pythagoras theorem

$b^2 = a^2 + c^2$

$= (6.92)^2 + (4)^2$

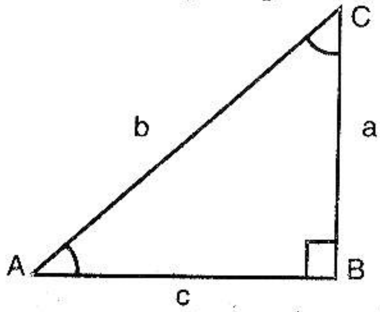
$= 47.8864 + 16$

$b^2 = 83.8864$

$b = 7.99cm = 8cm$

Q.8 a = 3cm, c = 4cm

Sol. Consider a right angled ΔABC.



Here

$a = 3cm$

$c = 4cm$

$m\angle B = 90^0$

$m\angle A = ?$

$m\angle C = ?$

$b = ?$

Now by Pythagoras theorem

$b^2 = a^2 + c^2$

$= (3)^2 + (4)^2$

$= 9 + 16$

$b^2 = 25$

$b = 5cm$

Now

$\sin m\angle A = \frac{a}{b}$

$\sin m\angle A = \frac{3}{5}$

$\Rightarrow m\angle A = Sin^{-1}\left(\frac{3}{5}\right) = 36.87^0$

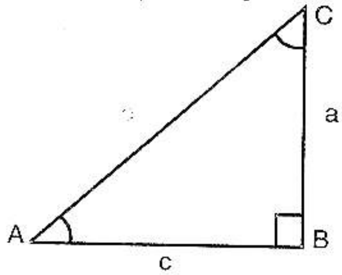
$m\angle C = 90^0 - m\angle A$

$= 90^0 - 36.87^0$

$m\angle C = 53.13^0$

Q.9 **b = 12cm , a = 6cm , $m\angle B = 90^0$**

Sol. Consider a right angled ΔABC.



Here

$$m\angle B = 90^0$$
$$b = 12cm$$
$$a = 6cm$$
$$m\angle A = ?$$
$$m\angle C = ?$$
$$c = ?$$

Now by Pythagoras theorem

$$c^2 = b^2 - a^2$$
$$= (12)^2 - (6)^2$$
$$c^2 = 144 - 36$$
$$c^2 = 108$$
$$c = \sqrt{108}$$
$$= \sqrt{36\times 3}$$
$$= \sqrt{36}\times\sqrt{3}$$
$$= 6\sqrt{3}\ cm$$

Now

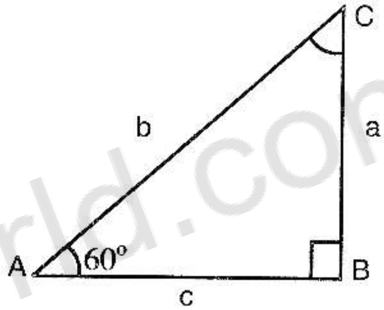
$$\sin m\angle A = \frac{a}{b}$$
$$= \frac{6}{12}$$
$$\sin m\angle A = \frac{1}{2}$$
$$m\angle A = \sin^{-1}\left(\frac{1}{2}\right)$$
$$\Rightarrow m\angle A = 30^0$$

Now

$$m\angle C = 90^0 - m\angle A$$
$$= 90^0 - 30^0$$
$$m\angle C = 60^0$$

Q.10 **$m\angle A = 60^0$, a = 6cm, $m\angle B = 90^0$**

Sol. Consider a right angled ΔABC.



Here

$$m\angle A = 60^0$$
$$a = 6cm$$
$$m\angle B = 90^0$$
$$m\angle C = ?$$
$$b = ?$$
$$c = ?$$
$$\sin m\angle A = \frac{a}{b}$$
$$\sin 60^0 = \frac{6}{b}$$
$$\frac{\sqrt{3}}{2} = \frac{6}{b}$$
$$b = \frac{6\times 2}{\sqrt{3}}$$
$$b = \frac{12}{\sqrt{3}}$$
$$= \frac{4\times 3}{\sqrt{3}}$$
$$b = 4\sqrt{3}cm$$
$$b = 6.92cm$$

Now by Pythagoras theorem

$c^2 = b^2 - a^2$

$= (4\sqrt{3})^2 - (6)^2$

$= (4)^2(\sqrt{3})^2 - (6)^2$

$= 16(3) - 36$

$= 48 - 36$

$c^2 = 12$

$\sqrt{c^2} = \sqrt{12}$

$c = \sqrt{4\times 3}$

$c = \sqrt{4}\times\sqrt{3}$

$c = 2\sqrt{3}\ cm$

Also

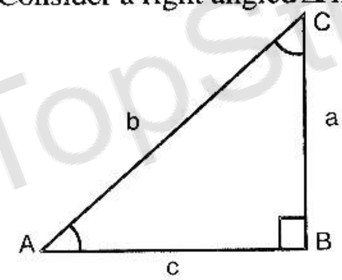
$m\angle C = 90^0 - m\angle A$

$= 90^0 - 60^0$

$m\angle C = 30^0$

Q.11 **$b = 5\sqrt{2}$ cm , $c = 5$ cm**

Sol. Consider a right angled ΔABC.



Here

$m\angle B = 90^0$

$b = 5\sqrt{2}cm$

$c = 5cm$

$m\angle A = ?$

$m\angle C = ?$

$a = ?$

$a^2 = b^2 - c^2$

$= (5\sqrt{2})^2 - (5)^2$

$a^2 = (5)^2(\sqrt{2})^2 - (5)^2$

$a = \sqrt{25(2) - 25}$

$a = \sqrt{50 - 25}$

$a = \sqrt{50-25}$

$a = \sqrt{25}$

$a = 5\ cm$

Now

$\tan m\angle A = \frac{a}{c}$

$= \frac{5}{5}$

$\tan m\angle A = 1$

$m\angle A = \tan^{-1}(1)$

So $m\angle A = 45^o$

Now

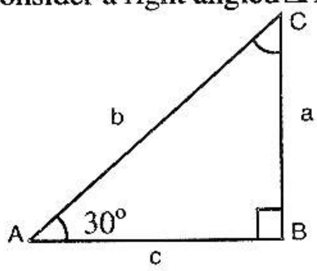
$m\angle C = 90^0 - m\angle A$

$= 90^0 - 45^0$

$m\angle C = 45^0$

Q.12 **$m\angle A = 30^o$, $b = 4\sqrt{3}$ cm, $m\angle B = 90^o$**

Sol. Consider a right angled ΔABC.



Here

$m\angle A = 30^0$

$b = 4\sqrt{3}cm$

$m\angle B = 90^0$

$m\angle C = ?$

$a = ?$

$c = ?$

$m\angle C = 90^0 - m\angle A$

$= 90^0 - 30^0$

$m\angle C = 60^0$

Now

$$\sin m\angle A = \frac{a}{b}$$

$$\sin 30^0 = \frac{a}{4\sqrt{3}}$$

$$\frac{1}{2} = \frac{a}{4\sqrt{3}}$$

$$\frac{a}{4\sqrt{3}} = \frac{1}{2}$$

$$a = 2\sqrt{3}cm$$

$$a = 3.46cm$$

Now, by Pythagoras theorem

$$c^2 = b^2 - a^2$$

$$= \left(4\sqrt{3}\right)^2 - \left(2\sqrt{3}\right)^2$$

$$= (4)^2(\sqrt{3})^2 - (2)^2(\sqrt{3})^2$$

$$= 16(3) - 4(3)$$

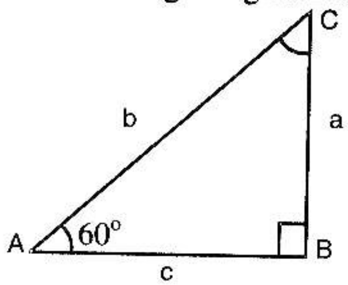
$$c^2 = 48 - 12$$

$$c^2 = 36$$

$$c = 6\ cm$$

Q.13 $m\angle A = 60^o$, $b = 4\sqrt{3}$ **cm,** $m\angle B = 90^o$

Sol. Consider a right angled ΔABC.



Here

$$m\angle A = 60^0$$

$$m\angle B = 90^0$$

$$b = 4\sqrt{3}cm$$

$$m\angle C = ?$$

$$a = ?$$

$$c = ?$$

$$m\angle C = 90^0 - m\angle A$$

$$= 90^0 - 60^0$$

$$m\angle C = 30^0$$

$$\sin m\angle A = \frac{a}{b}$$

$$\sin 60^0 = \frac{a}{4\sqrt{3}}$$

$$\frac{\sqrt{3}}{2} = \frac{a}{4\sqrt{3}}$$

$$\frac{a}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$a = \frac{4\sqrt{3}.\sqrt{3}}{2}$$

$$a = 2(\sqrt{3})^2$$

$$a = 2(3)$$

$$a = 2\times 3$$

$$a = 6cm$$

Now, by Pythagoras theorem

$$c^2 = b^2 - a^2$$

$$= \left(4\sqrt{3}\right)^2 - (6)^2$$

$$= (4)^2(\sqrt{3})^2 - 36$$

$$= 16(3) - 36$$

$$c^2 = 48 - 36$$

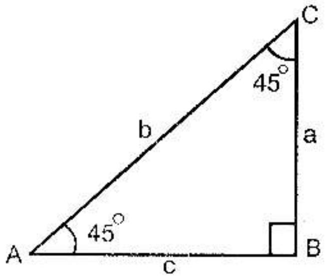
$$c^2 = 12$$

$$c = \sqrt{12}$$

$$c = 3.46\ cm$$

Q.14 $b = 4\sqrt{2}$ **cm,** $c = 4$ **cm,** $m\angle B = 90^o$

Sol. Consider a right angled ΔABC.



Here

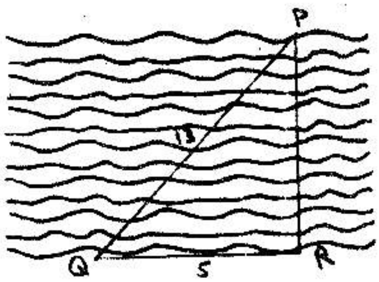
$$b = 4\sqrt{2}\,cm$$
$$c = 4cm$$
$$m\angle B = 90^0$$
$$a = ?$$
$$m\angle A = ?$$
$$m\angle C = ?$$
$$a^2 = b^2 - c^2$$
$$a^2 = \left(4\sqrt{2}\right)^2 - (4)^2$$
$$= (4)^2(\sqrt{2})^2 - 16$$
$$= 16(2) - 16$$
$$a^2 = 32 - 16$$
$$a^2 = 16$$
$$a = \sqrt{16}$$
$$a = 4\ cm$$

Now

$$\tan m\angle A = \frac{a}{c}$$
$$= \frac{4}{4}$$
$$\tan m\angle A = 1$$
$$m\angle A = \tan^{-1}(1)$$
$$\Rightarrow \quad m\angle A = 45^0$$
$$m\angle C = 90^0 - m\angle A$$
$$= 90^0 - 45^0$$
$$m\angle C = 45^0$$

Q.15 Let Q and R be the two point on the same bank of a canal. The point P is placed on the other bank straight to point R. Find the width of the canal.

Sol.



Here

$$m\,\overline{QR} = 5$$
$$m\overline{PQ} = 13$$

Width of Canal = w = $\overline{PR}$ = ?

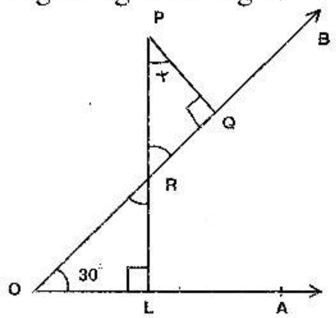
Applying Pythagoras theorem on ΔPQR

$$\left|\overline{PQ}\right|^2 = \left|\overline{QR}\right|^2 + \left|\overline{PR}\right|^2$$
$$\left|\overline{\mathrm{PR}}\right|^2 = \left|\overline{\mathrm{PQ}}\right|^2 - \left|\overline{\mathrm{QR}}\right|^2$$
$$\left|\overline{PR}\right|^2 = 13^2 - 5^2$$
$$= 169 - 25$$
$$\left|\overline{PR}\right|^2 = 144$$
$$\left|\overline{PR}\right| = \sqrt{144}$$
$$w = 12 \text{ unit}$$

So width of canal is 12 unit

Q.16 Find the measure of the angle $\Psi = \angle LPQ$ in the adjoining figure.

Sol: Consider the adjoining figure as OLR is a right angled triangle.



So

$m\angle ORL = 90^0 - 30^0$

$m\angle ORL = 60^0$

But $m\angle ORL = m\angle PRQ$ (vertical angles)

So

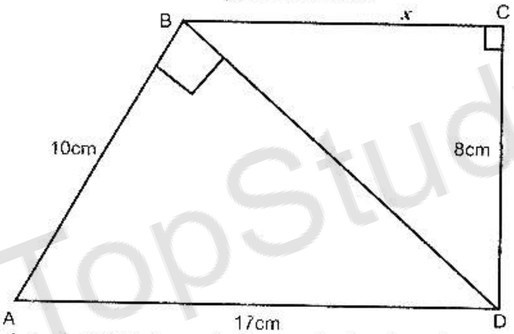
$m\angle PRQ = 60^0$

Now in right angled ΔPQR

$$m\angle LPQ = \psi$$
$$= 90^0 - m\angle PRQ$$
$$= 90^0 - 60^0$$
$$= 30^0$$

Q.17 Calculate the length x in adjoining figure.

Consider a fig as shown:



As ΔABD is a right angled triangle.

So

$$\left(\overline{BD}\right)^2 = \left(\overline{AD}\right)^2 - \left(\overline{AB}\right)^2$$
$$= (17)^2 - (10)^2$$
$$= 289 - 100$$
$$\left(\overline{BD}\right)^2 = 189$$
$$\Rightarrow \overline{BD} = \sqrt{189}$$

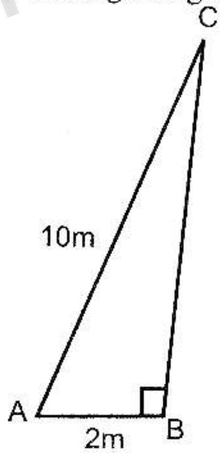
Now again in right angled ΔBCD

$$x^2 = \left(\overline{BD}\right)^2 - \left(\overline{CD}\right)^2$$
$$= \left(\sqrt{189}\right)^2 - (8)^2$$
$$= 189 - 64$$
$$x^2 = 125$$
$$x = \sqrt{125}$$
$$x = \sqrt{25 \times 5}$$
$$x = 5\sqrt{5}cm$$

Q.18 If the ladder is placed along the wall such that the foot of the ladder is 2m away from the wall. If the length of the ladder is 10m, find the height of the wall where ladder is placed?

Sol: Here AC is the ladder and BC be the wall then

ΔABC is a right angled triangle.



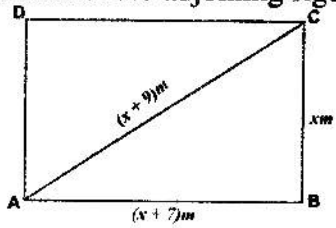
Here

$$\left(\overline{BC}\right)^2 = \left(\overline{AC}\right)^2 - \left(\overline{AB}\right)^2$$
$$= (10)^2 - (2)^2$$
$$= 100 - 4$$
$$\left(\overline{BC}\right)^2 = 96$$
$$\therefore \overline{BC} = \sqrt{96}$$

$= 4\sqrt{6}$ is height of wall.

Q.19 The diagonal of a rectangular field ABCD is (x + 9) m and the sides are (x+7)m and x m. Find the value of x

Sol. Consider the adjoining figure



Here

$\overline{AB} = (x+7)m$

$\overline{BC} = xm$

$\overline{AC} = (x+9)m$

As ABCD is a rectangular field
So ABC is a right angled triangle.
Hence by Pythagoras Theorem

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$

$$(x+9)^2 = (x+7)^2 + x^2$$

$\therefore\ (a+b)^2 = a^2 + 2ab + b^2$

$$x^2+18x+81 = x^2+14x+49+x^2$$

$$x^2+18x+81 = 2x^2+14x+49$$

$$2x^2+14x+49 = x^2+18x+81$$

$$2x^2+14x+49 - x^2 - 18x - 81 = 0$$

$$x^2-4x-32 = 0$$

$$x^2-8x+4x-32 = 0$$

$$x(x-8)+4(x-8) = 0$$

$$(x-8)(x+4) = 0$$

$\Rightarrow\ x = 8, -4$

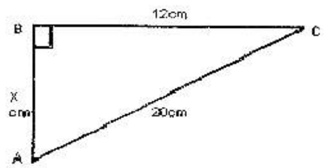
But as length cannot be negative
So

$x = 8$ m

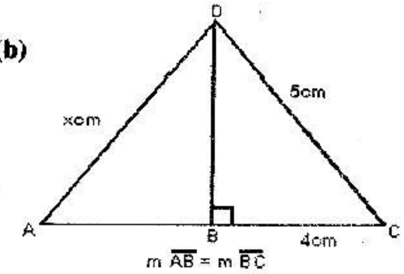
Hence height of wall is = 8m

Q.20 Calculate the value of 'x' in each case.

(a)



(b)



Sol: Consider fig (a)
As ABC is a right angled triangle.
So by Pythagoras Theorem

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$

$$(20)^2 = x^2 + (12)^2$$

$$400 = x^2 + 144$$

$$x^2 = 400 - 144$$

$$x^2 = 256$$

$$x = 16cm$$

Now Consider the fig (b)

As $m\overline{AB} = m\overline{BC}$

So $m\overline{AB} = 4cm$

Applying Pythagoras theorem on ΔBCD

$$(\overline{BD})^2 = (\overline{DC})^2 - (\overline{BC})^2$$

$$= (5)^2 - (4)^2$$

$$= 25 - 16$$

$$(\overline{BD})^2 = 9$$

So $\overline{BD} = 3$

$\therefore\ m\overline{BD} = 3cm$

Again applying Pythagoras theorem on ΔABD

$$(\overline{AD})^2 = (\overline{AB})^2 + (\overline{BD})^2$$

$$x^2 = (4)^2 + (3)^2$$

$$= 16 + 9$$

$$x^2 = 25$$

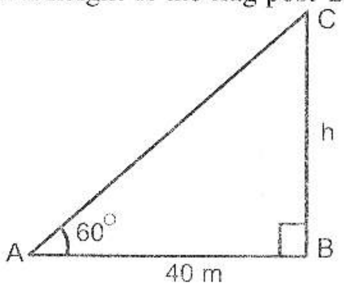
So

$x = 5cm$

EXERCISE 8.5

Q.1 The angle of elevation of the top of a flag post from a point on the ground level 40m away from the flat post is 60^o. Find the height of the post.

Sol. Let height of the flag post $\overline{BC} = h$.



Here

$\overline{AB} = 40m$

$m\angle BAC = 60^0$

From fig

$\frac{\overline{BC}}{\overline{AB}} = \tan 60^0$

$\frac{h}{40} = \sqrt{3}$

$h = 40\sqrt{3}$

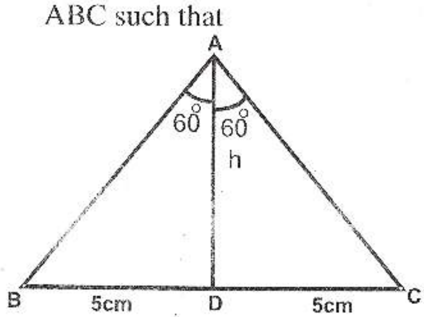
$= 40(1.732)$

$h = 69.2$

So height of flag post is 69.2m

Q.2 An isosceles triangle has a vertical angle of 120^o and base 10cm long. Find the length of its altitude.

Sol. Consider the isosceles triangle ABC such that



$\overline{AB} = \overline{AC}$

As $m\angle BAC = 120^0$

So $m\angle BAD = 60^0$

Here $\overline{BC} = 10cm$

So $\overline{BD} = \frac{1}{2}(10)$

$= 5cm$

Let h be the length of altitude

In ΔABD

$\frac{\overline{BD}}{\overline{AD}} = \tan 60^0$

$\frac{5}{h} = \sqrt{3}$

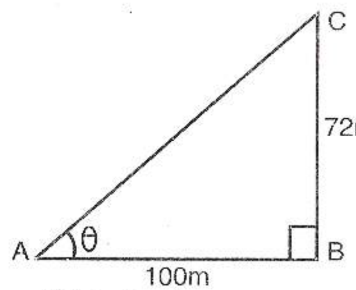
$h = \frac{5}{\sqrt{3}}$

$h = 2.89cm$

So length of altitude is 2.89cm

Q.3 A tree is 72m high. Find the angle of elevation of its top 100m away on the ground level.

Sol.



Here BC be the tree

$\overline{AB} = 100m$

Let θ be the angle of elevation of the top of tree.

In right angled ΔABC

$\tan\theta = \frac{\overline{BC}}{\overline{AB}}$

$$= \frac{72}{100}$$

$$\tan\theta = 0.72$$

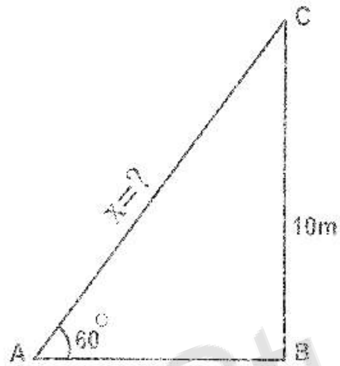
$$\theta = \tan^{-1}(0.72)$$

$$\theta = 35.75^0$$

So angle of elevation is 35.75^0

Q.4 A ladder makes an angle of 60^o with the ground and reaches a height of 10m along the wall. Find the length of the ladder.

Sol.



Here AC be the ladder and BC be the wall

Also $m\angle BAC = 60^0$

We want to find $\overline{AC}$

Let $\overline{AC} = x$

In right angled ΔABC

$$sin60^0 = \frac{\overline{BC}}{\overline{AC}}$$

$$\frac{\sqrt{3}}{2} = \frac{10}{x}$$

$$\sqrt{3}x = 2\times10$$

$$x = \frac{20}{\sqrt{3}}$$

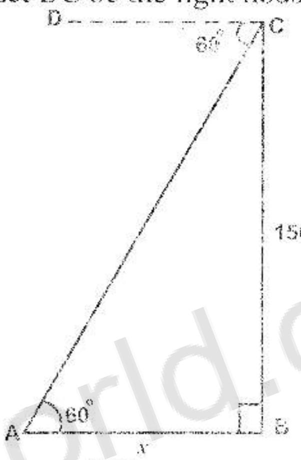
$$x = \frac{20}{1.732}$$

$$= 11.54m$$

So length of ladder is 11.54m

Q.5 A light house tower is 150m high from the sea level. The angle of depression from the top of the tower to a ship is 60^o. Find the distance between the ship and the tower.

Sol. Let BC be the light house tower



$$\overline{BC} = 150m$$

Here $m\angle DCA = 60^0$

So $m\angle BAC = 60^0$

Let A be the position of ship

Suppose

$$\overline{AB} = x$$

In the right angled ΔABC

$$\tan 60^0 = \frac{\overline{BC}}{\overline{AB}}$$

$$\sqrt{3} = \frac{150}{x}$$

$$x = \frac{150}{\sqrt{3}}$$

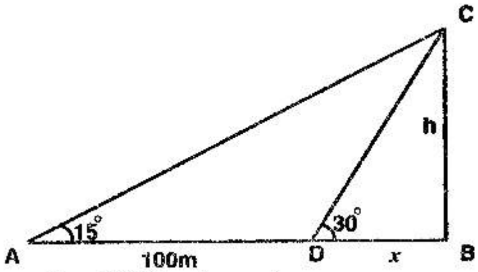
$$= \frac{150}{1.732}$$

$$= 86.6m$$

So required distance is 86.6m

Q.6 Measure of an angle of elevation of the top of a pole is 15^o from a point on the ground. In walking 100m towards the pole the measure of angle is found to be 30^o. Find the height of the pole.

Sol.



Let BC be the pole

Here

$$m\angle BAC = 15^0$$
$$m\angle BDA = 30^0$$

Let

$$\overline{BD} = x$$
$$\overline{BC} = h$$

In ΔBDC

$$\frac{h}{x} = \tan 30^0$$
$$\frac{h}{x} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow x = \sqrt{3}h$$

In ΔABC

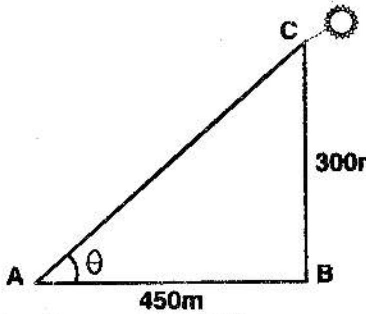
$$\frac{h}{100+x} = \tan 15^0$$

$\left(\text{Put value of x} = \sqrt{3}h\right)$

$$\frac{h}{100+\sqrt{3}h} = 0.27$$
$$h = 27+0.48h$$
$$h-0.48h = 27$$
$$h(1-0.48) = 27$$
$$0.52h = 27$$
$$h = 50m$$

Q.7 Find the measure of an angle of elevation of the Sun, if a tower 300m high casts a shadow 450m long.

Sol.



Consider the tower as BC.

Here $\overline{AB} = 450m$

$\overline{BC} = 300m$

Let θ be the angle of elevation of the sun.

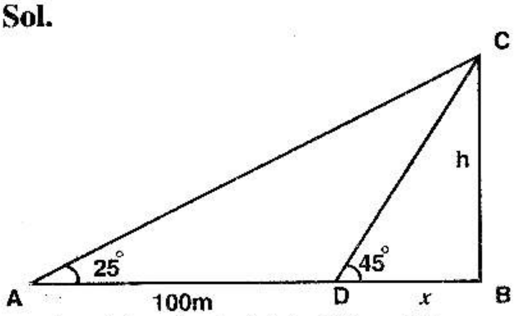
In ΔABC

$$\tan\theta = \frac{\overline{BC}}{\overline{AB}}$$
$$= \frac{300}{450}$$
$$= \frac{30}{45}$$
$$\tan\theta = \frac{2}{3}$$
$$\tan\theta = 0.6667$$
$$\theta = \tan^{-1}(0.6667)$$
$$\Rightarrow \theta = 33.66^0$$

So angle of elevation of the sun is 33.66^0

Q.8 Measure of angle of elevation of the top of a cliff is 25°, on walking 100 meters towards the cliff, measure of angle of elevation of the top is 45°. Find the height of the cliff.

Sol.



Let h be the height of the cliff.

Here $\overline{AD} = 100m$

Suppose

$$\overline{BD} = x$$

In ΔBCD

$$\tan 45^0 = \frac{h}{x}$$

$$1 = \frac{h}{x}$$

$$x = h$$

In ΔABC

$$\frac{h}{100+x} = \tan 25^0$$

$$\frac{h}{100+h} = 0.4663$$

$$h = 46.63 + 0.4663h$$

$$h - 0.4663h = 46.63$$

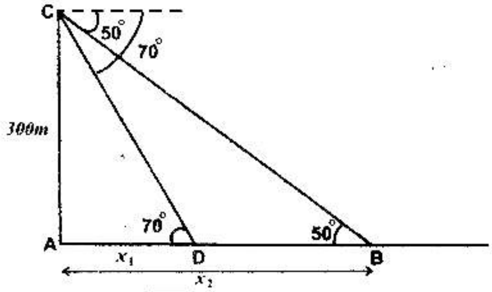
$$h(1-0.4663) = 46.63$$

$$0.5337h = 46.63$$

$$h = 87.3m$$

Q.9 From the top of a hill 300m high, the measure of the angle of depression of a point on the nearer shore of the river is 70° and measure of the angle of depression of a point directly across the river is 50°. Find the width of the river. How far is the river from the foot of the hill?

Sol.



Let $\overline{AD} = x_1$

$\overline{AB} = x_2$

Then

Width of river $= x_2 - x_1$

In ΔACD

$$\tan 70^0 = \frac{300}{x_1}$$

$$x_1 = \frac{300}{\tan 70^0}$$

$$= \frac{300}{2.74}$$

$$x_1 = 109.19$$

Now in ΔABC

$$\tan 50^0 = \frac{300}{x_2}$$

$$x_2 = \frac{300}{\tan 50^0}$$

$$= \frac{300}{1.19}$$

$$x_2 = 251.72$$

So width of river $= x_2 - x_1$

$= 251.72 - 109.19$

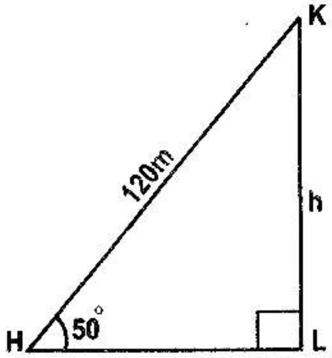
$= 142.6m$

the distance of river from foot of hill is

$= x_1 = 109.19m$

Q.10 A kite has 120m of string attached to it when an elevation of 50°. How far it is above the hand holding it? (Assume that the string is taut).

Sol.



Let H be the hand and K be the position of kite.

Here

$\overline{HK} = 120m$

$m\angle LHK = 50^0$

Let h be the height of kite above the hand

In right angled ΔHKL

$$Sin50^0 = \frac{\overline{LK}}{\overline{HK}}$$

$$0.766 = \frac{h}{120}$$

or $h = 120 \times 0.766$

$h = 91.92m$

So height of kite above hand is 91.92m

EXAMPLES

Example 1: Show that $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Solution: L.H.S $= (\sec^2\theta - 1)\cos^2\theta$

$= \tan^2\theta.\cos^2\theta$

$(\because 1 + \tan^2\theta = \sec^2\theta)$

$= \dfrac{\sin^2\theta}{\cos^2\theta}.\cos^2\theta$

$\left(\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right)$

$= \sin^2\theta$

= R.H.S

Hence $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Example 2:

Show that $(\tan\theta + \cot\theta) = \sec\theta\ \text{cosec}\theta$

Sol: L.H.S $= \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta.\sin\theta + \cos\theta\cos\theta}{\cos\theta.\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta.\sin\theta}$

$= \dfrac{1}{\cos\theta.\sin\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \dfrac{1}{\cos\theta}.\dfrac{1}{\sin\theta}$

$= \sec\theta.\text{cosec}\theta$

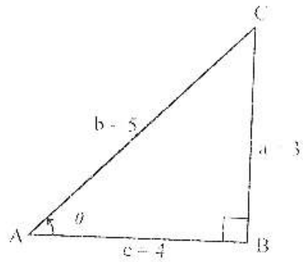
= R.H.S

Hence $\tan\theta + \cot\theta = \sec\theta\ \text{cosec}\theta$

Example 3:

If $\tan\theta = \dfrac{3}{4}$, find the remainin trigonometric radios, when θ lies in th first quadrant.

Sol:



Given $\tan\theta = \dfrac{3}{4} = \dfrac{a}{c}$

Where a = 3 , c = 4

By Pythagoras theorem, we have

$b^2 = a^2 + c^2$

$= 3^2 + 4^2$

$= 9 + 16$

$= 25$

$\Rightarrow b = 5$

Therefore,

$\sin\theta = \dfrac{a}{b} = \dfrac{3}{5}$: $\text{cos}ec\theta = \dfrac{b}{a} = \dfrac{5}{3}$

$\cos\theta = \dfrac{c}{b} = \dfrac{4}{5}$: $\sec\theta = \dfrac{b}{c} = \dfrac{5}{3}$

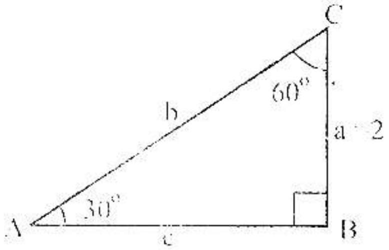
$\cot\theta = \dfrac{c}{a} = \dfrac{4}{3}$

Example 4:

Solve triangle ABC in which

$m\angle B = 90^\circ$, $m\angle A = 30^\circ$, $a = 2$

Sol:



We are required to find b, c and $m\angle C$

Now $m\angle C = m\angle B - m\angle A$

$= 90^\circ - 30^\circ$

$= 60^\circ$ (i)

and $\dfrac{a}{b} = \sin 30^\circ$

or $\dfrac{2}{b} = \sin 30^\circ$ ($a = 2$)

or $\dfrac{2}{b} = \dfrac{1}{2}$ $\left(\therefore \sin 30^\circ = \dfrac{1}{2}\right)$

or $b = 4$ (ii)

and $\dfrac{a}{c} = \tan 30^\circ$

or $\dfrac{2}{c} = \dfrac{1}{\sqrt{3}}$ $\left(\therefore a = 2, \tan 30^\circ = \dfrac{1}{\sqrt{3}}\right)$

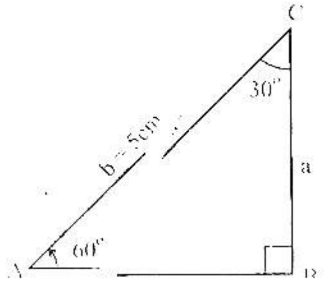
thus $c = 2\sqrt{3}$ (iii)

Hence (i), (ii) and (iii) give the required results.

Example 5:

Solve triangle ABC, when $m\angle A = 60^\circ$, $b = 5cm$, $m\angle B = 90^\circ$

Sol:



We are required to find a, c and $m\angle C$

$m\angle A = 60^\circ$

$m\angle B = 90^\circ$

$m\angle C = 90^\circ - m\angle A$

$= 90^\circ - 60^\circ$

$= 30^\circ$ (i) Now $\dfrac{a}{b} = \sin 60^\circ$

or

$\dfrac{a}{5} = \dfrac{\sqrt{3}}{2}$ $\left(\therefore b = 5cm, \sin 60^\circ = \dfrac{\sqrt{3}}{2}\right)$

or $a = \dfrac{5\sqrt{3}}{2}$

or $a = 4.33$ cm (ii)

and $\dfrac{c}{b} = \cos 60^\circ$

or

$\dfrac{c}{5} = \dfrac{1}{2}$ $\left(\therefore b = 5cm, \cos 60^\circ = \dfrac{1}{2}\right)$

or $c = \dfrac{5}{2}$

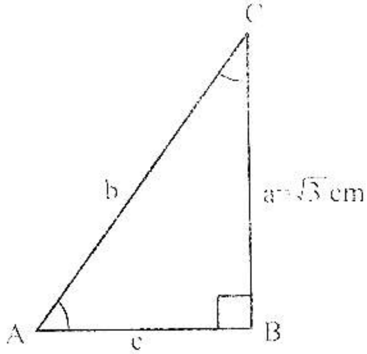
or $c = 2.5$ cm (iii)

Equations (i), (ii) and (iii) give the required results.

Example 6:

Solve triangle ABC, when $a = \sqrt{3}cm$

$c = 1cm$ and $m\angle B = 90^\circ$



Sol:

We are required to find b, $m\angle A$, $m\angle C$

by Pythagoras theorem, we have

$b^2 = c^2 + a^2$

or $b^2 = (1)^2 + (\sqrt{3})^2$ (Putting Value)

or $b^2 = 1 + 3$

or $b^2 = 4$

or $b^2 = 2$ (i)

Now $\sin m\angle A = \dfrac{a}{b}$

$= \dfrac{\sqrt{3}}{2}$

or $m\angle A = 60^\circ$ (ii)

and $m\angle C = 90^\circ - m\angle A$

$= 90 - 60^\circ$

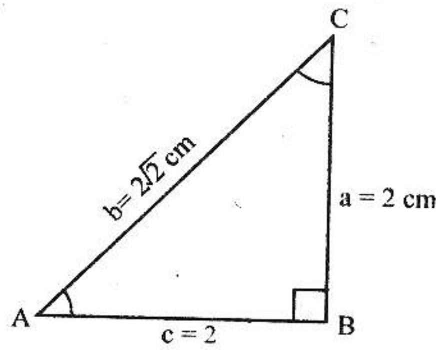
$= 30^\circ$

Equations (i), (ii) and (iii) give the required results.

Example 7:

Solve triangle ABC, when $a = 2\text{cm}$, $b = 2\sqrt{2}$ cm and $m\angle B = 90^\circ$

Sol:



We are required to find $m\angle A$, $m\angle C$.

By Pythagoras theorem, we have

$b^2 = c^2 + a^2$

or $c^2 = b^2 - a^2$

$= (2\sqrt{2})^2 - (2)^2$

$= 8 - 4$

$= 4$

or $c = 2$ cm (i)

Now $\dfrac{c}{b} = \cos m\angle A$

or $\cos m\angle A = \dfrac{c}{b} = \dfrac{1}{\sqrt{2}}$ cm

$\Rightarrow m\angle A = 45^\circ$ (ii)

Thus $m\angle C = 90^\circ - m\angle A$

$= 90^\circ - 45^\circ$

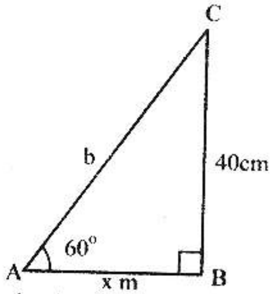
$m\angle A = 45^\circ$ (iii)

Equations (i), (ii) and (iii) give the required results.

Example 8:

The angle of elevation of the top of pole 40m high is 60° when seen from a point on the ground level. Find the distance of the point from the foot of the pole.

Sol:



In the triangle ABC, we have

$m\overline{BC} = 40\text{m}$

Distance from the pole $= m\overline{AB} = x$ m

$m\angle A = 60^\circ$

$m\overline{BC} = 40$ m

In the right angled triangle ABC,

$\tan 60^\circ = \dfrac{m\overline{BC}}{m\overline{AB}}$

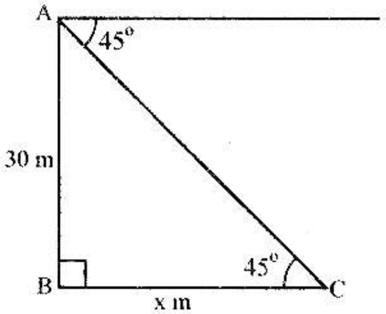
or $\sqrt{3} = \dfrac{40}{x}$

or $x = \frac{40}{\sqrt{3}}$

or $x = 23.23$ m

Example 9: From the top of a lookout tower, the angle of depression of a building on the ground level is of 45^o. How far is the building from the foot of the tower, if the height of the tower is 30m?

Sol:



In right triangle ABC, we have

$m\overline{AB} = 30m$

$m\angle A = m\angle C = 45^o$

$m\overline{BC} = x$

$\tan 45^o = \frac{m\overline{AB}}{m\overline{BC}}$

or $1 = \frac{30}{x}$

or $x = 30m$

Objective

Q.1. Four answers of each item are given from which only one is true. Tick the correct answer.

1. How many elements has a triangle?
(a) two (b) three
(c) six (d) five

2. The word trigonometry means:
(a) triangle measurement
(b) square measurement
(c) earth measurement
(d) circle measurement

3. In a right angled triangle one angle is of measure.
(a) 90^0 (b) 30^0
(c) 60^0 (d) 60^0

4. Side opposite to right-angle is
(a) perpendicular (b) hypotenuse
(c) base
(d) perpendicular base

5. Sin θ = ______
(a) $\frac{\text{Hypotenuse}}{\text{Perpendicular}}$ (b) $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$
(c) $\frac{\text{Base}}{\text{Hypotenuse}}$ (d) $\frac{\text{Perpendicular}}{\text{Base}}$

6. Cos θ = ______
(a) $\frac{\text{Base}}{\text{Hypotenuse}}$ (b) $\frac{\text{Hypotenuse}}{\text{Base}}$
(c) $\frac{\text{Perpendicular}}{\text{Base}}$ (d) $\frac{\text{Base}}{\text{Perpendicular}}$

7. Tan θ = ______
(a) $\frac{\text{Perpendicular}}{\text{Base}}$ (b) $\frac{\text{Base}}{\text{Perpendicular}}$
(c) $\frac{\text{Hypotenuse}}{\text{Base}}$ (d) $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$

8. Cosec θ = ____
(a) $\frac{\text{Hypotenuse}}{\text{Base}}$ (b) $\frac{\text{Hypotenuse}}{\text{Perpendicular}}$
(c) $\frac{\text{Base}}{\text{Hypotenuse}}$ (d) $\frac{\text{Base}}{\text{Perpendicular}}$

9. Sec θ = ______

(a) $\dfrac{\text{Hypotenuse}}{\text{Base}}$ (b) $\dfrac{\text{Base}}{\text{Perpendicular}}$

(c) $\dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$ (d) $\dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

10. Cot θ = ______

(a) $\dfrac{\text{Base}}{\text{Hypotenuse}}$ (b) $\dfrac{\text{Base}}{\text{Perpendicular}}$

(c) $\dfrac{\text{Hypotenuse}}{\text{Base}}$ (d) $\dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

11. In a right-angled triangle,

$(\text{Perpendicular})^2 + (\text{Base})^2 =$ ____

(a) $(\text{Hypotenuse})^2$ (b) Hypotenuse

(c) $\sqrt{\text{Hypotenuse}}$

(d) $(\text{Hypotenuse})^3$

12. In a right angled isosceles triangle, number of equal sides are

(a) 1 (b) 2

(c) 3 (d) 4

13. Cosec θ = ______

(a) $\dfrac{1}{Cos\ \theta}$ (b) $\dfrac{1}{Sin\ \theta}$

(c) $\dfrac{1}{Tan\ \theta}$ (d) $\dfrac{1}{Sec\ \theta}$

14. Sec θ = ______

(a) $\dfrac{1}{Sin\ \theta}$ (b) $\dfrac{1}{Cot\ \theta}$

(c) $\dfrac{1}{Cos\ \theta}$ (d) $\dfrac{1}{Tan\ \theta}$

15. Cot θ = ______

(a) $\dfrac{1}{Sin\ \theta}$ (b) $\dfrac{1}{Tan\ \theta}$

(c) $\dfrac{1}{Cos\ \theta}$ (d) $\dfrac{1}{\text{Cosec}\ \theta}$

16. Tan θ = ______

(a) $\dfrac{Sin\,\theta}{Cos\,\theta}$ (b) $\dfrac{Cos\,\theta}{Sin\,\theta}$

(c) $\dfrac{1}{\text{Cos}\ \theta}$ (d) $\dfrac{1}{\text{Cosec}\ \theta}$

17. Cot θ = ______

(a) $\dfrac{Sin\,\theta}{Cos\,\theta}$ (b) $\dfrac{Cos\,\theta}{Sin\,\theta}$

(c) $\dfrac{1}{Co\sec\ \theta}$ (d) $\dfrac{1}{Cos\ \theta}$

18. $\text{Sin}\,(90^0 - \theta) =$ ______

(a) Cos θ (b) Tan θ

(c) Sin θ (d) Cot θ

19. $\text{Tan}\,(90^0 - \theta) =$ ______

(a) Sin θ (b) Sec θ

(c) Cot θ (d) Cosec θ

20. $\text{Cot}\,(90^0 - \theta) =$ ______

(a) Tan θ (b) Cot θ

(c) Sin θ (d) Cosec θ

21. $\text{Sec}\,(90^0 - \theta) =$ ______

(a) Tan θ (b) Cosec θ

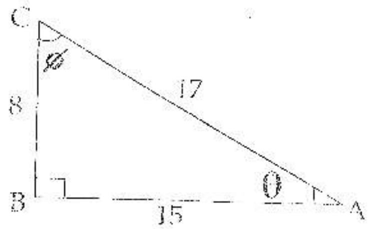
(c) Sec θ (d) Cot θ

22. $\text{Cosec}\,(90^0 - \theta) =$ ______

(a) Sec θ (b) Cot θ

(c) Sin θ (d) Tan θ

23. In a right angled Δ A B C , Sin ∅ = ____



(a) $\dfrac{17}{8}$ (b) $\dfrac{8}{17}$

(c) $\dfrac{15}{17}$ (d) $\dfrac{17}{15}$

24. Sin θ Cosec θ = ______

(a) Sin θ (b) 1

(c) Cos θ (d) Cosec θ

25. Cos θ Sec θ = ______

(a) Sin θ (b) Cos θ

(c) 1 (d) Cosec θ

26. Tan θ Cot θ = ______

(a) 1 (b) $\sin\theta$

(c) Cos θ (d) Cosec θ

27. $\tan(90^0 - 45^0) =$ ______

(a) Cot 45^0 (b) Cot 30^0

(c) Cos 45^0 (d) Sin 45^0

28. $\cos(90^0 - 45^0) =$ ______

(a) Sin 60^0 (b) Cos30^0

(c) Sin 45^0 (d) Cot 30^0

29. $Sin^2\theta + Cos^2\theta =$ ____

(a) 1 (b) 2

(c) 0 (d) 3

30. $1 + Cot^2\theta =$ ______

(a) $\tan^2\theta$ (b) $\operatorname{Cosec}^2\theta$

(c) $\sec^2\theta$ (d) 1

31. $1 + \tan^2\theta =$ ______

(a) $\sec^2\theta$ (b) $\operatorname{Cosec}^2\theta$

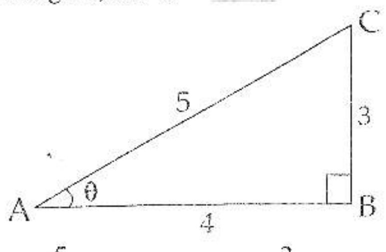
(c) $\dfrac{\tan^2\theta}{\cos^2\theta}$ (d) 1

32. $\dfrac{\cos\theta}{\sin\theta} =$ ______

(a) $\dfrac{1}{\cot\theta}$ (b) $\dfrac{1}{\tan\theta}$

(c) $\dfrac{1}{\sin\theta}$ (d) $\dfrac{1}{\operatorname{Cosec}\theta}$

33. In figure, Sin θ = ______



(a) $\dfrac{5}{3}$ (b) $\dfrac{3}{5}$

(c) $\dfrac{5}{4}$ (d) $\dfrac{4}{3}$

34. Sec θ = ______

(a) $1 + \tan^2\theta$ (b) $\sqrt{1 + \tan^2\theta}$

(c) $1 - \tan^2\theta$ (d) $\operatorname{Cosec}^2\theta$

35. Cosec θ = ____

(a) $\sqrt{1 + \cot^2\theta}$ (b) $1 + \cot^2\theta$

(c) $1 - \cot^2\theta$ (d) $\sqrt{1 + \tan^2\theta}$

36. Cot θ Sin θ = ____

(a) $\dfrac{1}{\sin\theta}$ (b) $\dfrac{1}{\sec\theta}$

(c) $\dfrac{1}{\operatorname{Cosec}\theta}$ (d) $\dfrac{1}{\cos\theta}$

37. Tan 45^0 = ______

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 1 (d) $\dfrac{1}{2}$

38. Cot 60^0 = ______

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $\dfrac{2}{\sqrt{3}}$

39. Sin 30^0 = ______

(a) $\dfrac{1}{2}$ (b) $\sqrt{3}$

(c) $\dfrac{2}{\sqrt{3}}$ (d) $\dfrac{1}{\sqrt{3}}$

40. 2 Sin 45^0 + 2 Cos 45^0 = ______

(Lahore Board 2010)

(a) $\dfrac{4}{\sqrt{2}}$ (b) $\dfrac{2}{\sqrt{2}}$

(c) $\dfrac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

41. 5 Cos 45^0 + 3 Sin 45^0 = ______

(a) $4\sqrt{2}$ (b) $\dfrac{1}{\sqrt{2}}$

(c) $\frac{4}{\sqrt{2}}$ (d) $\sqrt{2}$

42. Tan $30^0 =$ ______

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{3}{\sqrt{2}}$

43. $\text{Tan}\,30^0\ \text{Cot}\,30^0 + 1 =$ ____ *(Lahore Board 2010)*

(a) 1 (b) 2

(c) $\sqrt{3}$ (d) $1+\sqrt{3}$

44. The value of cos 30^0 is ____.

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

45. The value of sec 30^0 is ____.

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

46. The value of cosec 30^0 is ____.

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2

47. The value of cot 30^0 is ____.

(a) $\sqrt{3}$ (b) $\frac{1}{2}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{1}{\sqrt{2}}$

48. The value of sin 60^0 is ____.

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2}$ (d) $\sqrt{3}$

49. The value of cos 60^0 is ____.

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2

50. The value of tan 60^0 is ____.

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{2}$

51. The value of sec 60^0 is ____.

(a) $\frac{1}{2}$ (b) 2

(c) $\sqrt{2}$ (d) $\sqrt{\frac{3}{2}}$

52. The value of cosec 60^0 is ____.

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{2}}$

53. The value of sin 45^0 is ____.

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

54. The value of cos 45^0 is ____.

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

55. The value of cot 45^0 is ____.

(a) $\sqrt{2}$ (b) 1

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{3}$

56. The value of sec 45^0 is ____.

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

57. The value of cosec 45^0 is ______.

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

58. $2 \sin 60^0 \cos 60^0 =$ _____.

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{2}$

59. $3 \cos 45^0 + 4 \sin 45^0 =$ _____.

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{7}{\sqrt{2}}$

(c) $\frac{\sqrt{2}}{7}$ (d) $\frac{2}{\sqrt{3}}$

60. $\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0 =$ ______.

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{4}$

61. $\cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0 =$ _____.

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{2}{\sqrt{3}}$

62. $5 \cos 45^0 - 3 \sin 45^0 =$ _____.

(a) $\frac{2}{\sqrt{2}}$ (b) $\frac{4}{\sqrt{2}}$

(c) $\frac{3}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

63. $(1 + \sin\theta)(1 - \sin\theta) =$ ______.

(a) $\frac{1}{\sec^2\theta}$ (b) $\tan^2\theta$

(c) $\text{cosec}^2\theta$ (d) $\cot^2\theta$

64. $(1 - \cos\theta)(1 + \cos\theta) =$ ______.

(a) $\sin^2\theta$ (b) $\cos^2\theta$

(c) $\tan^2\theta$ (d) $\text{cosec}^2\theta$

65. $(\sec\theta - 1)(\sec\theta + 1) =$ ______.

(a) $\tan^2\theta$ (b) $\cot^2\theta$

(c) $\sin^2\theta$ (d) $\cos^2\theta$

66. $\tan\theta + \cot\theta =$ ____.

(a) $\sin\theta \cos\theta$ (b) $\text{cosec}\,\theta \sec\theta$

(c) $\cot\theta$ (d) $\tan\theta\ \text{cosec}\,\theta$

67. $\cot\theta \,.\, \sin\theta =$ _____.

(a) $\frac{1}{\sec\theta}$ (b) $\text{cosec}\,\theta$

(c) $\tan\theta$ (d) $\cot\theta$

68. $(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1) =$ ______.

(a) $\cot^2\theta$ (b) $\sec^2\theta$

(c) $\tan^2\theta$ (d) $\sin^2\theta$

69. $\frac{\cos\theta}{\sin\theta} =$ ______.

(a) $\tan\theta$ (b) $\frac{1}{\tan\theta}$

(c) $\text{cosec}\,\theta$ (d) $\sec\theta$

70. $\sin\theta\,(\text{cosec}\,\theta - \sin\theta) =$ _______.

(a) $\frac{1}{\sec^2\theta}$ (b) $\tan^2\theta$

(c) $\text{cosec}^2\theta$ (d) $\cot^2\theta$

71. $(\sec^2\theta - 1)\cos^2\theta =$ ______.

(a) $\sin^2\theta$ (b) $\cos^2\theta$

(c) $\cot^2\theta$ (d) $\text{cosec}^2\theta$

72. $\tan\theta + \cot\theta =$ ____.

(a) $\sec\theta$ (b) $\sec\theta\ \text{cosec}\,\theta$

(c) $\text{cosec}\,\theta$ (d) $\tan\theta \cot\theta$

73. The reciprocal of $\sin m\angle A$ is ______.

(a) $\cos m\angle A$ (b) $\text{cosec}\ m\angle A$

(c) $\sec m\angle A$ (d) $\cot m\angle A$

74. $\sin^2 m\angle B + \cos^2 m\angle B =$ ______.

(a) 1 (b) 2

(c) 3 (d) 4

75. $\sin^2 60^0 + \cos^2 60^0 =$ ______.

(a) 2

(b) 3

(c) 4

(d) 1

76. If base and height of a right angled triangle are 2cm each. Then Hypotenuse is ______.
(a) $2\sqrt{2}$ (b) $2\sqrt{3}$
(c) 8 (d) 12

77. $\sqrt{1-\sin^2 m<B} =$ ____.
(a) cos m∠B (b) cosec m∠B
(c) sec m∠B (d) cot m∠B

78. If $\sin\theta = \frac{1}{2}$ then $\theta =$ ______.
(a) 30^0 (b) 60^0
(c) 45^0 (d) 90^0

79. If $\sin\theta = \frac{\sqrt{3}}{2}$ then $\theta =$ ______.
(a) 90^0 (b) 60^0
(c) 45^0 (d) 30^0

80. If $\cos\theta = \frac{\sqrt{3}}{2}$ then $\theta =$ ______.
(a) 30^0 (b) 60^0
(c) 45^0 (d) 30^0

81. If $\tan\theta = 1$ then $\theta =$ ______.
(a) 45^0 (b) 30^0
(c) 60^0 (d) 90^0

82. If $\cot\theta = \frac{1}{\sqrt{3}}$ then $\theta =$ ______.
(a) 60^0 (b) 30^0
(c) 45^0 (d) 90^0

83. Tan 40^0 = cot ______.
(a) 40^0
(b) 60^0
(c) 50^0
(d) 30^0

84. Cos 80^0 = sin ______.
(a) 10 (b) 20
(c) 100 (d) 5

85. Cot 67^0 = tan ______.
(a) 23^0 (b) 40^0
(c) 30^0 (d) 20^0

86. If $\text{cosec}\,\theta = 2$ then $\theta =$ ____.
(a) 30^0 (b) 60^0
(c) 90^0 (d) 45^0

87. $\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta} =$ ____.
(a) $\text{cosec}^2\theta$ (b) $\text{cosec}\,\theta$
(c) $\cot\theta$ (d) $\cot^2\theta$

88. $\frac{\tan\theta}{\cot\theta} =$ ______.
(a) 1 (b) $\sin\theta$
(c) $\sin^2\theta$ (d) $\tan^2\theta$

89. $\sin^2 45^0 \cos^2 45^0 =$ ______.
(a) 1 (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) 2

90. $\frac{1}{\tan 30^0} =$ ______.
(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

ANSWERS

1.	c	**2.**	a	**3.**	a	**4.**	b	**5.**	b	**6.**	a	**7.**	a
8.	b	**9.**	a	**10.**	b	**11.**	a	**12.**	b	**13.**	b	**14.**	c
15.	b	**16.**	a	**17.**	b	**18.**	a	**19.**	c	**20.**	a	**21.**	b
22.	a	**23.**	c	**24.**	b	**25.**	c	**26.**	a	**27.**	a	**28.**	c
29.	a	**30.**	b	**31.**	a	**32.**	b	**33.**	b	**34.**	b	**35.**	a
36.	b	**37.**	c	**38.**	b	**39.**	a	**40.**	a	**41.**	a	**42.**	c
43.	b	**44.**	c	**45.**	a	**46.**	d	**47.**	a	**48.**	a	**49.**	b
50.	b	**51.**	b	**52.**	b	**53.**	a	**54.**	b	**55.**	b	**56.**	a
57.	b	**58.**	a	**59.**	b	**60.**	b	**61.**	a	**62.**	a	**63.**	a
64.	a	**65.**	a	**66.**	b	**67.**	a	**68.**	a	**69.**	b	**70.**	a
71.	a	**72.**	b	**73.**	b	**74.**	a	**75.**	d	**76.**	a	**77.**	a
78.	a	**79.**	b	**80.**	a	**81.**	a	**82.**	d	**83.**	c	**84.**	a
85.	a	**86.**	a	**87.**	a	**88.**	d	**89.**	b	**90.**	b		